REFRACTION - EXAMPLES AND APPLICATION

1. WATER WAVES REFRACTION
2. FIBRE OPTICS
3. THIN LENS ANALYSIS AND RAY DIAGRAMS

DIFFRACTION:

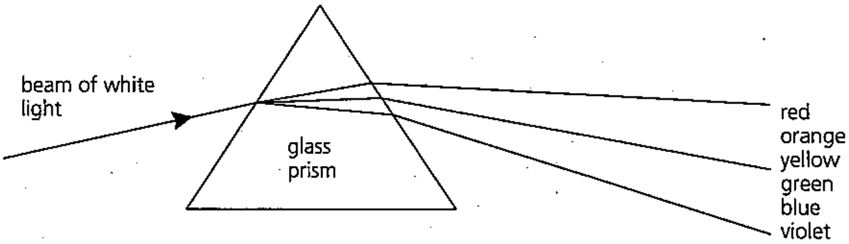
BENDING OF WAVES AROUND AN OBSTRUCTION OR OPENING WHEN WAVELENGTH OF WAVE IS SIMILAR TO OBSTRUCTION OR OPENING SIZE.

Dispersion

When waves pass through some materials the velocity of the wave may depend slightly on the wavelength of the disturbance. This effect is called **dispersion.**

Water, plastics and glass are all materials that disperse electromagnetic waves. Generally, shorter wavelengths travel at slightly lower wave velocities than do longer wavelengths. When light is passed through a prism a spectrum of colours is produced. This occurs because the velocity of light passing through a material depends upon the wavelength of the light. The longer wavelengths (red end of spectrum) are refracted less by the prism than the shorter wavelengths (violet end of spectrum). This is shown in Figure 8.33.

Figure 8.33
Dispersion of light by a prism



Dispersion can be observed with a slinky spring. A pulse applied to a slinky fixed at both ends is observed to spread out slowly. Sound waves have little dispersion in air but are dispersed in water.

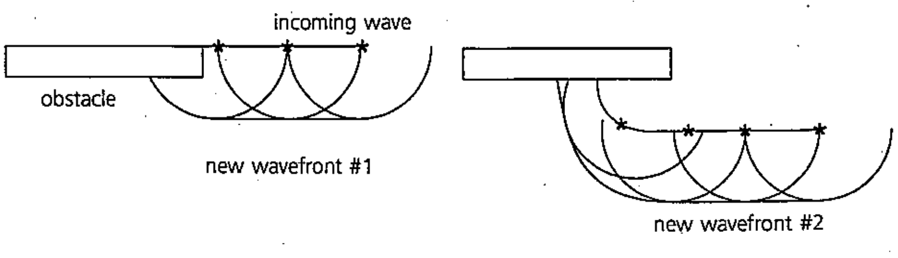
Diffraction

When parallel wavefronts meet an obstruction they are seen to bend around it. If they find a hole, they spread out in all directions after passing through the opening. As the opening is reduced in size, the wavefronts are seen to become almost circular. This effect, in which waves appear to bend around obstacles, is an example of **diffraction.**

Sound waves can therefore carry energy into areas behind obstacles which would normally be in shadow because light waves cannot bend far enough around the object. If someone shouts at you and throws a ball as you duck down behind a wall, you won't be hit by the ball, but you will still hear the shout. The sound waves diffract around the edges of the wall. The idea of diffraction follows closely from Huygens' wavefront principle. As the plane wave reaches the **aperture** (opening), each point on the wavefront at the aperture (1) can be thought of as producing a secondary circular wavelet. The envelope of these wavelets produces the wavefront at a later time. The wavelets add to produce a plane wavefront except near the edges of the obstacle where the wavefront curves around the obstacle's edge (Figure 8.34).

Diffraction effects are also visible behind slits, which are effectively two obstacles separated by a small distance. As the obstacles become closer, the slit narrows until only a small section of wavefront needs to be considered as a source of secondary wavelets. The envelope and hence the new wavefront become more nearly circular.

Figure 8.34
Diffraction of a wave at an edge



The effects due to diffraction become obvious only when the size of the hole, or the obstacle, is similar to the wavelength of the disturbance. Sound waves have wavelengths ranging from a few centimetres to several metres, so that they are diffracted by doors and windows. We can hear around corners. Light waves have wavelengths of about 10^{-7} m. They are not diffracted by apertures the size of doors. We cannot see around corners.

Figure 8.35 shows what happens to waves when slits and obstacles are larger than the wave's wavelength and when they are comparable to it.

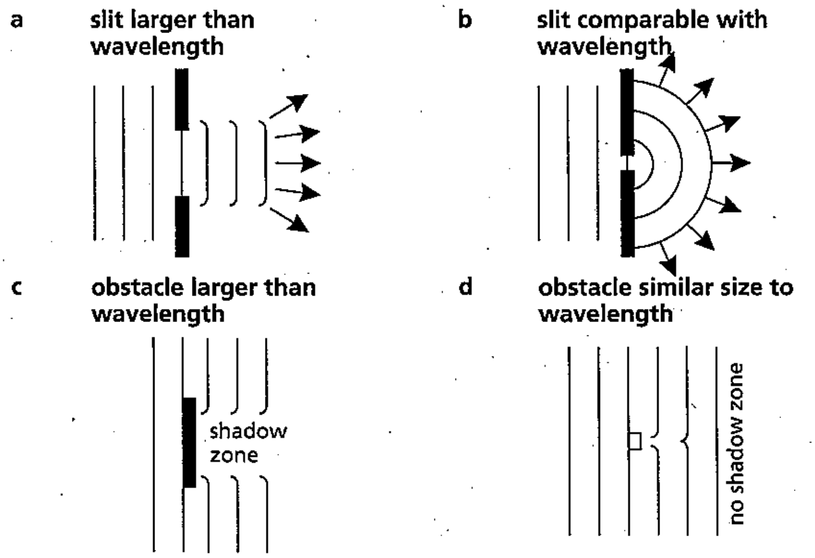


Figure 8.35
Diffraction effects are most obvious when the size of the hole, or the obstacle, is similar to the wavelength of the disturbance. Diffraction by an obstacle much larger than the wavelength leaves a shadow zone. The wave can be diffracted around a smaller obstacle

Interference of two-dimensional waves

It was shown earlier that waves can be added together in one dimension constructively or destructively. These effects are important in two-dimensional or three-dimensional waves. If two sources produce waves which move through a region, there will be positions where the disturbances will add constructively and others where they add destructively.

If the two sources:
- are point sources radiating circular waves
- keep in step or in phase
- produce waves of equal amplitude

then the pattern of constructive and destructive interference regions may be simply represented as shown in Figure 8.36.

Consider a ripple tank in which the vibrator is replaced by two balls connected to a single vibrating bar. The balls touching the surface of the water in the tank act as wave sources which are in phase. Each ball is responsible for a set of spherical waves which move outwards from the ball. Two sets of circular waves thus move across the surface of the water in the tank. If we take a snapshot of the resulting interference pattern, we find that there are positions where the waves cancel each other out and other positions where they interfere constructively and reinforce each other. If a second snapshot is taken shortly after, it is found that constructive and destructive interference occurs at exactly the same positions as in the previous snapshot. Since the two sources are in phase, the interference pattern does not change with time.

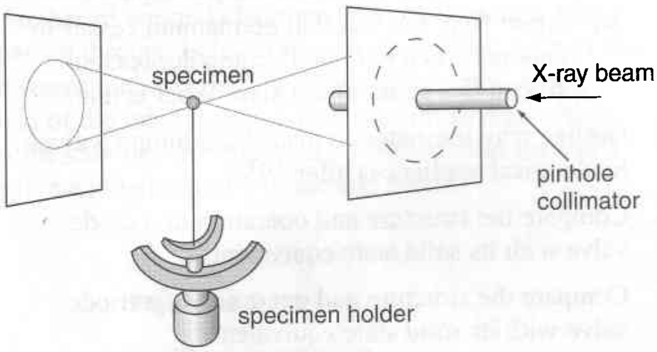
It is clear from Figure 8.36 that the two waves add constructively along a line joining points which are an equal distance from both sources. Many other lines link adjacent regions of constructive interference, where the maximum disturbances occur.

24 William Henry Bragg and William Lawrence Bragg

In 1912, two Australian Physicists, Sir William Henry Bragg and his 22 year old son Lawrence discussed research done by the German Physicist, Max von Laue, who used recently discovered X-rays to produce diffraction patterns from thin slivers of crystalline substances.

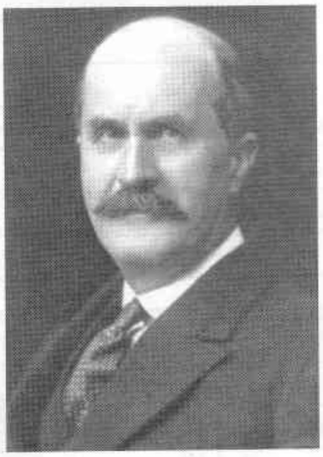
Lawrence Bragg proposed that the diffraction of the X-rays was caused by atoms in the crystal structure and that an analysis of the diffraction pattern would give clues as to the crystal structure of the substance.

Figure 24.1



The discovery of X-rays in 1895 by Wilhelm Roentgen was fortuitous for the Braggs. Diffraction patterns only occur if the gap through which waves travel is about the same size (or smaller) than the wavelength of those waves. Having a wavelength of around 10^{-10} m, X-rays were about the same size as the distance between atoms in crystals – hence the diffraction patterns by von Laue.

William Lawrence Bragg

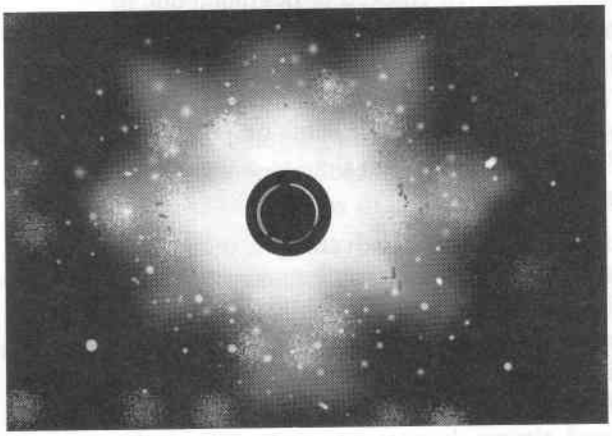


William Henry Bragg

The experiments by the Braggs were quite simple. X-rays were bounced off metal surfaces and the diffraction patterns formed by the reflected rays caught on photographic film (Figure 24.2). The pattern of dots indicated the positions of the atoms in the metal crystals.

The Braggs showed that atoms in metal crystals are so close together that their valence electron orbits overlap.

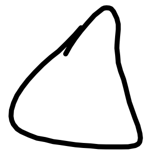
Figure 24.2



This means the valence electrons 'lose' their atomic identity – they do not belong to specific atoms. From this the **free electron model** for the structure of metals developed. This model regards metal crystal lattices as positively charged nuclei floating in a 'sea' of electrons. It is these electrons which give metals their high electrical conductivity. Scientists used the free electron model of metals to explain why their resistance increased when they were heated, and predicted the existence of superconductors at very low temperatures.

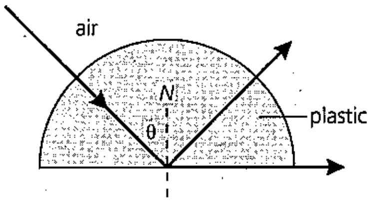
For You To Do

1. Why was the discovery of X-rays important for the Braggs' work?
2. Why would the use of visible light in their spectrometer not have given the Bragg's the results they wanted?
3. Assess the major impacts of the Braggs' work on science.
4. Contrast the free electron model and existing models for the structure of matter.
5. How does the free electron model account for the electrical conductivity of metals compared to other substances?
6. Define diffraction.
7. Outline the method used by the Braggs to determine the structure of crystals.



DISPERSION:

BREAKING UP OF
WHITE LIGHT INTO ITS SPECTRUM
OF COLOURS DUE TO SLIGHT DIFFERENCES
IN VELOCITY/REFRACTION AS IT
TRAVELS THROUGH A PRISM



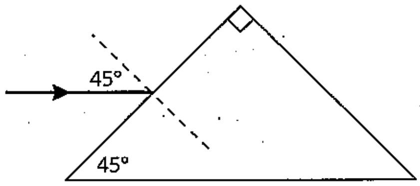
The refractive index of the glass for the yellow light is 1.517. Determine the value of the angle θ.

6 A ray of light from a ray box is directed at a semicircular plastic block. At the critical angle the refracted ray runs along the surface of the plastic as shown at left. The refractive index of the plastic is 1.40. Calculate the critical angle θ.

7 A girl stands 1.2 m from a fish tank and a fish is 0.30 m from the glass side of the tank, as shown in the diagram.

a What is the apparent distance apart as seen by the girl?
b What is the apparent distance apart as seen by the fish?

8 Complete the path of the light beam through the glass prism below. The prism has a refractive index of 1.50. Calculate the angles the beam makes at all air–glass interfaces.



12.4 Dispersion

When light waves pass through some materials, the velocity of the wave may depend slightly on the wavelength of the disturbance. This effect is called **dispersion.**

Water, plastics and glass are all materials that disperse electromagnetic waves. Generally, shorter wavelengths travel at slightly lower wave velocities than do longer wavelengths.

Newton first noticed in 1666 that, when white light is passed through a prism, a spectrum of colours is produced. This occurs because the velocity of light passing through a material depends upon the wavelength of the light. White light is in fact made up of different colours, each with their own wavelength. The longer wavelengths (red end of spectrum) are refracted less by the prism than the shorter wavelengths (violet end of spectrum; Figure 12.13). Table 12.3 lists the characteristics of the different colours.

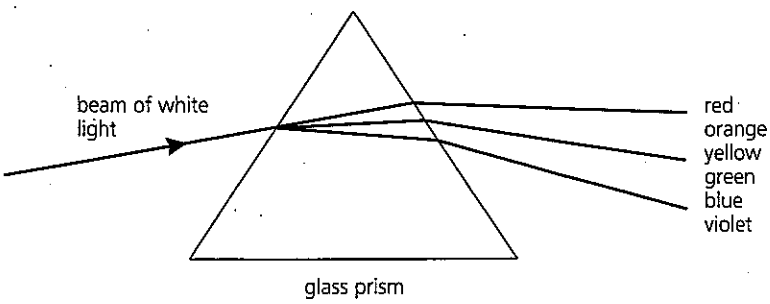


Figure 12.13 The dispersion of light produces a spectrum

Table 12.3 Characteristics of coloured light

Colour	Wavelength (nm)	Velocity	Refractive index*
red	670	0.661 *c*	1.514
yellow	590	0.659 *c*	1.517
green	550	0.658 *c*	1.519
blue	490	0.657 *c*	1.523
violet	405	0.653 *c*	1.532

* In crown glass. *Note*: $c = 3 \times 10^8$ m s^{-1}.

A double rainbow, formed by refraction and reflection in raindrops. The colours in the secondary rainbow are in reverse order to those in the primary rainbow.
Source: Bureau of Meteorology

Spectrometers

A study of the spectra emitted by gases and solids shows that different gases produce spectra with different features. The features in a spectrum provide a unique signature for the gas, allowing the composition of mixtures of gases to be identified accurately. Detailed study of a spectrum will also reveal the temperature of the emitting source, the pressure of the gases, and its velocity relative to the observer. The development of sensitive instruments to make observations of the spectra of sources is therefore important to scientific endeavours.

The simplest form of **spectrometer** is made up of the following components:

- a slit, which is illuminated by the light to be studied
- a collimating lens which makes the light beam plane parallel
- a dispersing element, often a glass prism
- a camera lens to focus the spectrum
- a detector to record the spectrum.

Figure 12.14 illustrates a simple spectrometer.

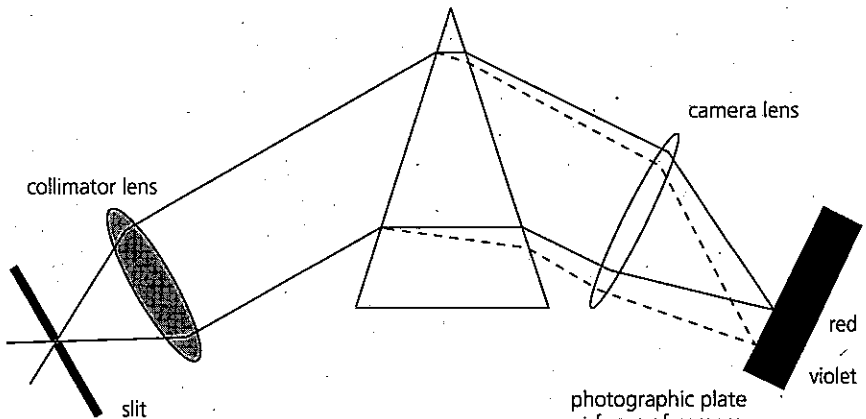


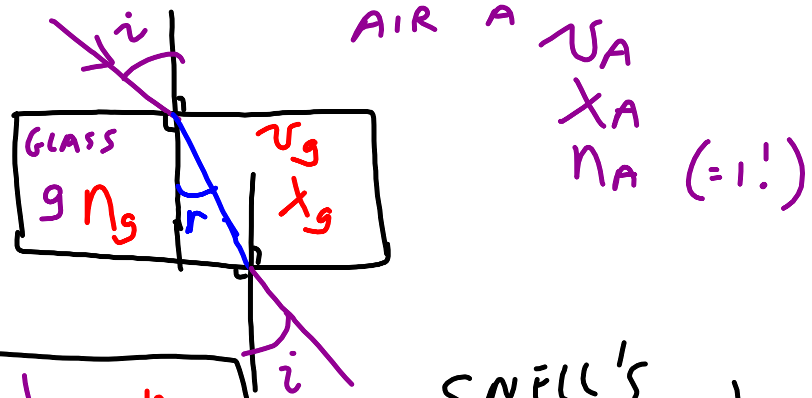
Figure 12.14
The spectrometer

12.5 Interference Effects

In chapter 8 it was seen that two waves will add together to produce a final disturbance which is the sum of the individual wave amplitudes. Light is no different from other waveforms in this respect. It too is associated with interference effects.

When light waves from two sources pass through the same region of space, the waves will add together. A detector placed anywhere in the region at any given moment will detect an electromagnetic disturbance with an amplitude which is the sum of the amplitude of the two waves at that point.

SO FINALLY WE CAN LINK ALL THESE POINTS INTO ONE SNELL'S EQUATION



$$\frac{\sin i}{\sin r} = \frac{v_A}{v_g} = \frac{\lambda_A}{\lambda_g} = \frac{n_g}{n_A}$$

← SNELL'S COMPLETE LAW !

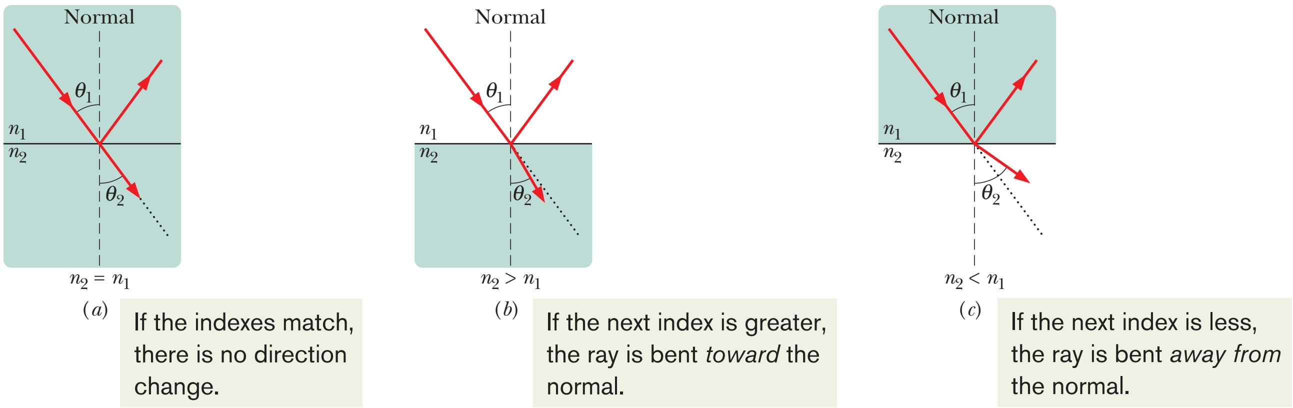


Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2. (*a*) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (*b*) toward the normal when $n_2 > n_1$ and (*c*) away from the normal when $n_2 < n_1$.

We can rearrange Eq. 33-40 as

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 \tag{33-41}$$

to compare the angle of refraction θ_2 with the angle of incidence θ_1. We can then see that the relative value of θ_2 depends on the relative values of n_2 and n_1:

1. If n_2 is equal to n_1, then θ_2 is equal to θ_1 and refraction does not bend the light beam, which continues in the *undeflected direction,* as in Fig. 33-17*a*.
2. If n_2 is greater than n_1, then θ_2 is less than θ_1. In this case, refraction bends the light beam away from the undeflected direction and toward the normal, as in Fig. 33-17*b*.
3. If n_2 is less than n_1, then θ_2 is greater than θ_1. In this case, refraction bends the light beam away from the undeflected direction and away from the normal, as in Fig. 33-17*c*.

Refraction *cannot* bend a beam so much that the refracted ray is on the same side of the normal as the incident ray.

Chromatic Dispersion

The index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light. The dependence of n on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction. This spreading of light is called **chromatic dispersion,** in which "chromatic" refers to the colors associated with the individual wavelengths and "dispersion" refers to the spreading of the light according to its wavelengths or colors. The refractions of Figs. 33-16 and 33-17 do not show chromatic dispersion because the beams are *monochromatic* (of a single wavelength or color).

Generally, the index of refraction of a given medium is *greater* for a shorter wavelength (corresponding to, say, blue light) than for a longer wavelength (say, red light). As an example, Fig. 33-18 shows how the index of refraction of fused quartz depends on the wavelength of light. Such dependence means that when a beam made up of waves of both blue and red light is refracted through a surface, such as from air into quartz or vice versa, the blue *component* (the ray corresponding to the wave of blue light) bends more than the red component.

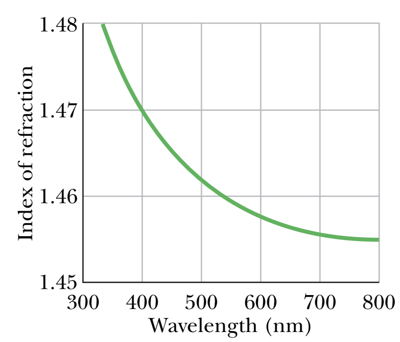


Fig. 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

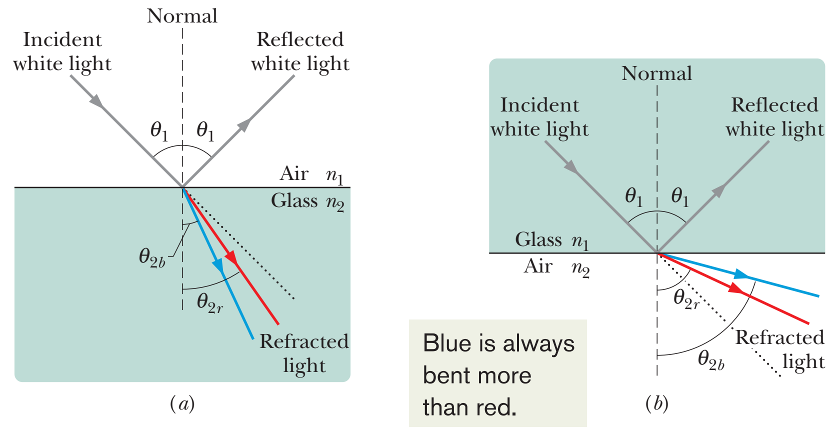


Fig. 33-19 Chromatic dispersion of white light. The blue component is bent more than the red component. (*a*) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (*b*) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.

A beam of *white light* consists of components of all (or nearly all) the colors in the visible spectrum with approximately uniform intensities. When you see such a beam, you perceive white rather than the individual colors. In Fig. 33-19*a*, a beam of white light in air is incident on a glass surface. (Because the pages of this book are white, a beam of white light is represented with a gray ray here. Also, a beam of monochromatic light is generally represented with a red ray.) Of the refracted light in Fig. 33-19*a*, only the red and blue components are shown. Because the blue component is bent more than the red component, the angle of refraction θ_{2b} for the blue component is *smaller* than the angle of refraction θ_{2r} for the red component. (Remember, angles are measured relative to the normal.) In Fig. 33-19*b*, a ray of white light in glass is incident on a glass–air interface. Again, the blue component is bent more than the red component, but now θ_{2b} is greater than θ_{2r}.

To increase the color separation, we can use a solid glass prism with a triangular cross section, as in Fig. 33-20*a*. The dispersion at the first surface (on the left in Figs. 33-20*a*, *b*) is then enhanced by the dispersion at the second surface.

Rainbows

The most charming example of chromatic dispersion is a rainbow. When sunlight (which consists of all visible colors) is intercepted by a falling raindrop,

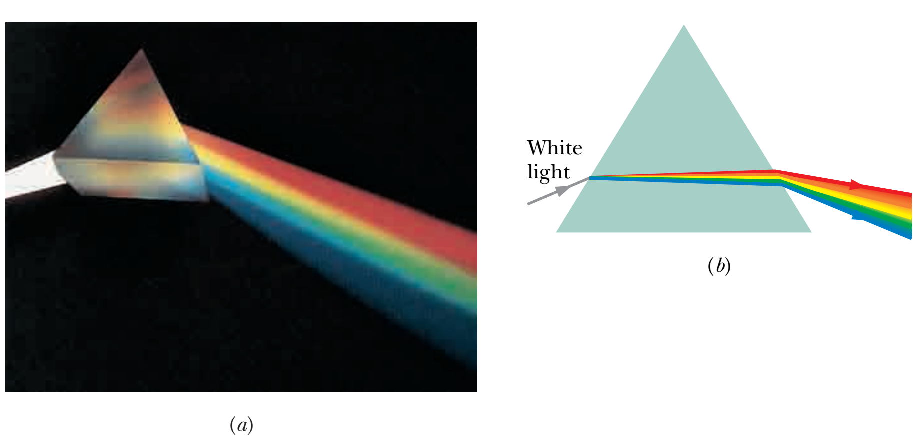
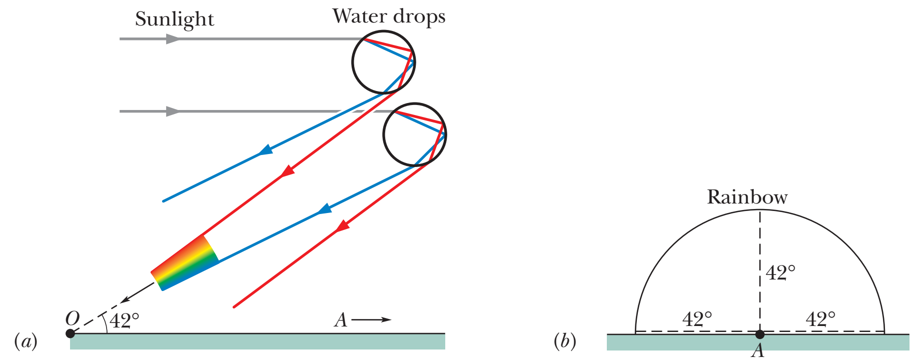


Fig. 33-20 (*a*) A triangular prism separating white light into its component colors. (*b*) Chromatic dispersion occurs at the first surface and is increased at the second surface. *(Courtesy Bausch & Lomb)*



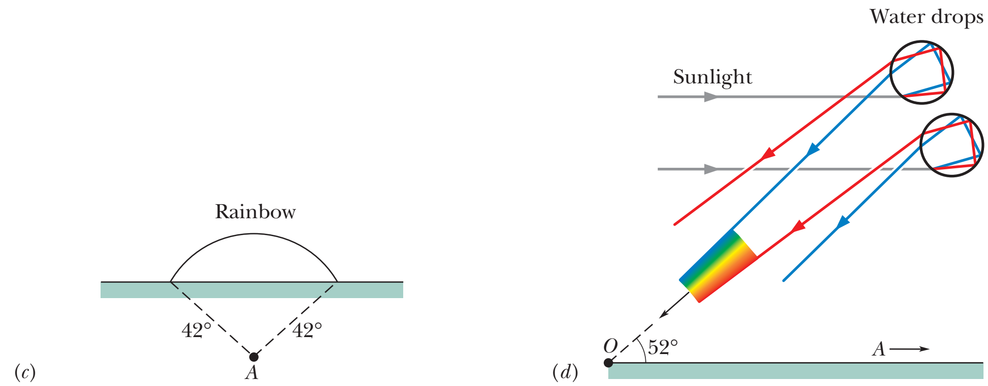


Fig. 33-21 (*a*) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point *A* is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of *A*. (*b*) Drops at 42° from *A* in any direction can contribute to the rainbow. (*c*) The rainbow arc when the Sun is higher (and thus *A* is lower). (*d*) The separation of colors leading to a secondary rainbow.

some of the light refracts into the drop, reflects once from the drop's inner surface, and then refracts out of the drop. Figure 33-21*a* shows the situation when the Sun is on the horizon at the left (and thus when the rays of sunlight are horizontal). The first refraction separates the sunlight into its component colors, and the second refraction increases the separation. (Only the red and blue rays are shown in the figure.) If many falling drops are brightly illuminated, you can see the separated colors they produce when the drops are at an angle of 42° from the direction of the *antisolar point A*, the point directly opposite the Sun in your view.

To locate the drops, face away from the Sun and point both arms directly away from the Sun, toward the shadow of your head. Then move your right arm directly up, directly rightward, or in any intermediate direction until the angle between your arms is 42°. If illuminated drops happen to be in the direction of your right arm, you see color in that direction.

Because any drop at an angle of 42° in any direction from *A* can contribute to the rainbow, the rainbow is always a 42° circular arc around *A* (Fig. 33-21*b*) and the top of a rainbow is never more than 42° above the horizon. When the Sun is above the horizon, the direction of *A* is below the horizon, and only a shorter, lower rainbow arc is possible (Fig. 33-21*c*).

Because rainbows formed in this way involve one reflection of light inside each drop, they are often called *primary rainbows.* A *secondary rainbow* involves two reflections inside a drop, as shown in Fig. 33-21*d*. Colors appear in the secondary rainbow at an angle of 52° from the direction of *A*. A secondary rainbow is wider and dimmer than a primary rainbow and thus is more difficult to see. Also, the order of colors in a secondary rainbow is reversed from the order in a primary rainbow, as you can see by comparing parts *a* and *d* of Fig. 33-21.

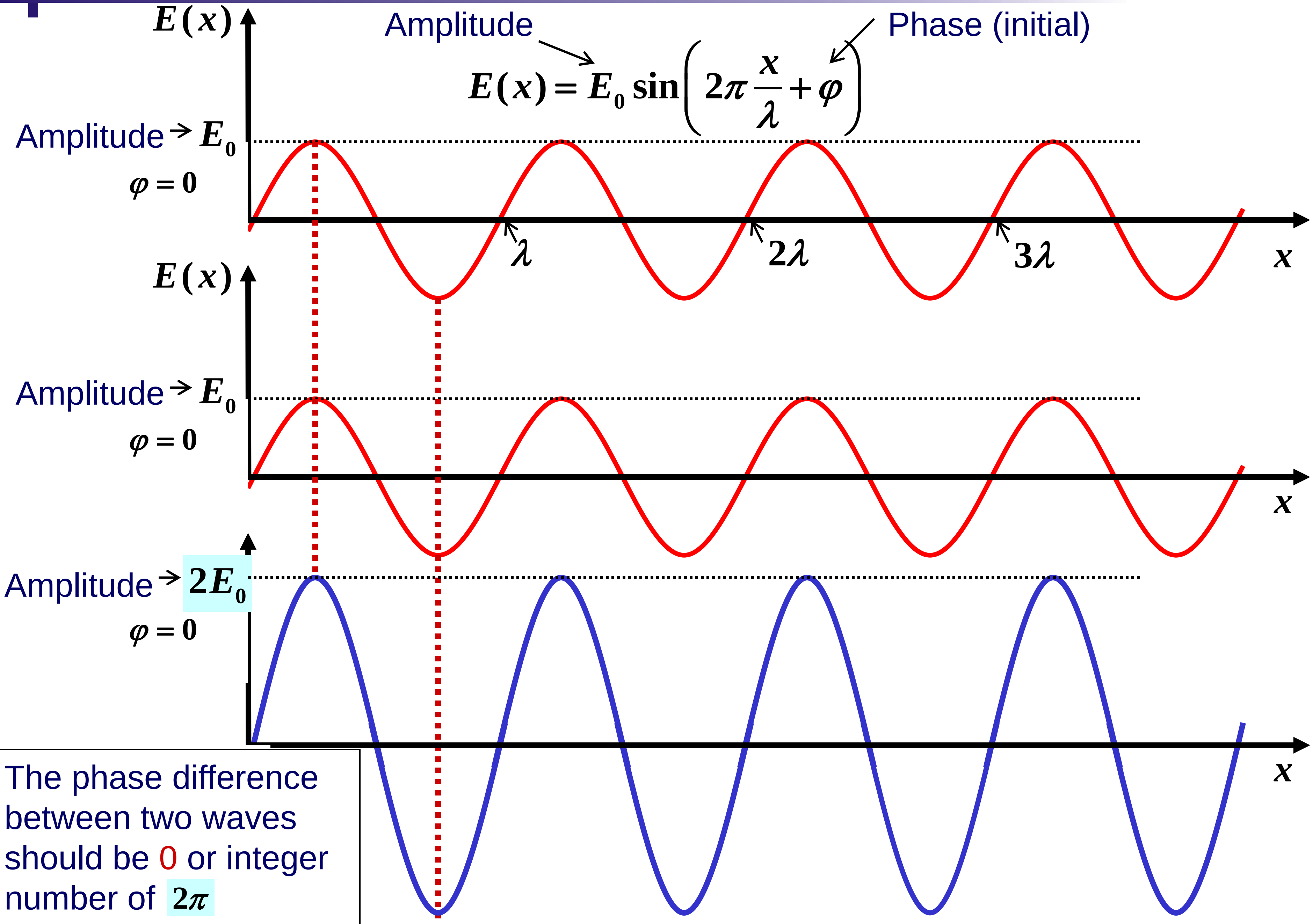
Rainbows involving three or four reflections occur in the direction of the Sun and cannot be seen against the glare of sunshine in that part of the sky. Rainbows involving even more reflections inside the drops are too dim to see.

INTERFERENCE:

CONSTRUCTIVE AND DESTRUCTIVE PROPERTY OF LIGHT DUE TO ADDING/SUBTRACTING OF CRESTS/TROUGHS.

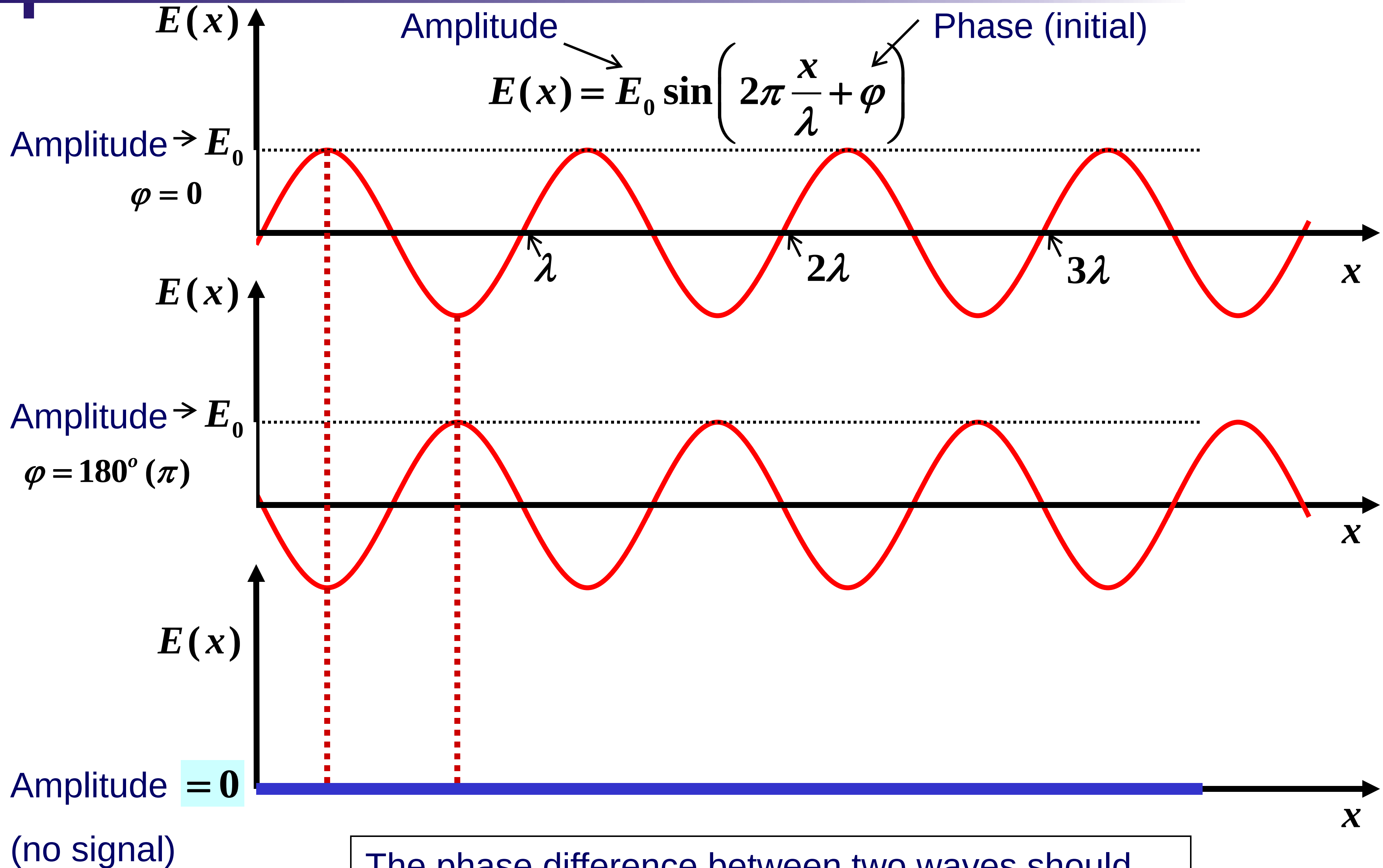
Light as a Wave
- Interference Effects

Sin-function: Constructive Interference



The phase difference between two waves should be 0 or integer number of 2π

Sin-function: Destructive Interference



The phase difference between two waves should be π or π plus integer number of 2π

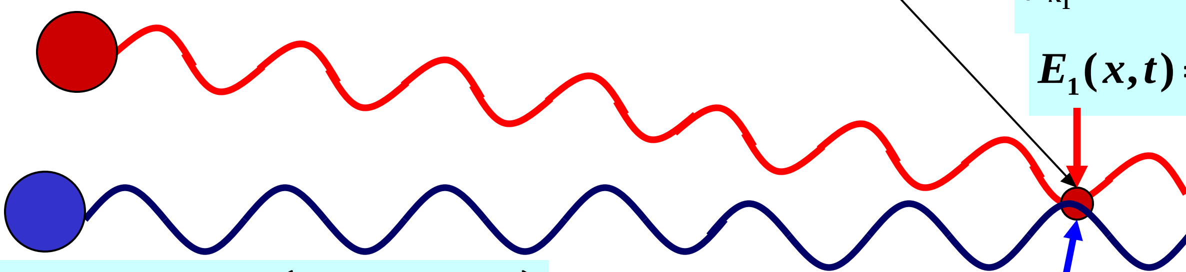
Waves: Interference

$$E_1(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi f t\right)$$

Interference – sum of two waves

$$\varphi_{x_1} = 2\pi \frac{x_1}{\lambda}$$

$$E_1(x,t) = E_0 \sin\left(2\pi f t + \varphi_{x_1}\right)$$



$$E_2(x,t) = E_0 \sin\left(2\pi \frac{x}{\lambda} + 2\pi f t\right)$$

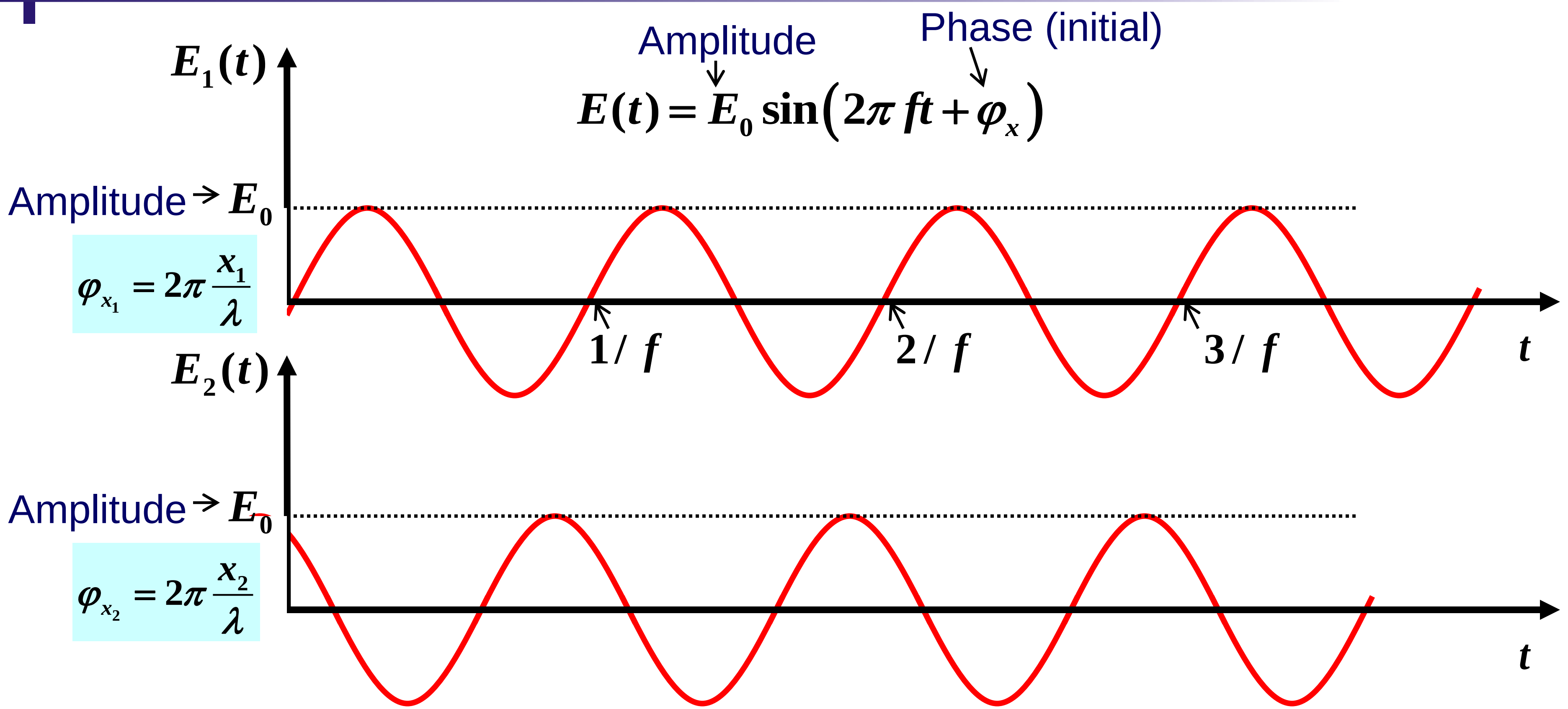
$$E_2(x,t) = E_0 \sin\left(2\pi f t + \varphi_{x_2}\right)$$

$$\varphi_{x_2} = 2\pi \frac{x_2}{\lambda}$$

- In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave

- In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave

Waves: Interference



Constructive Interference: The phase difference between two waves should be 0 or integer number of 2π

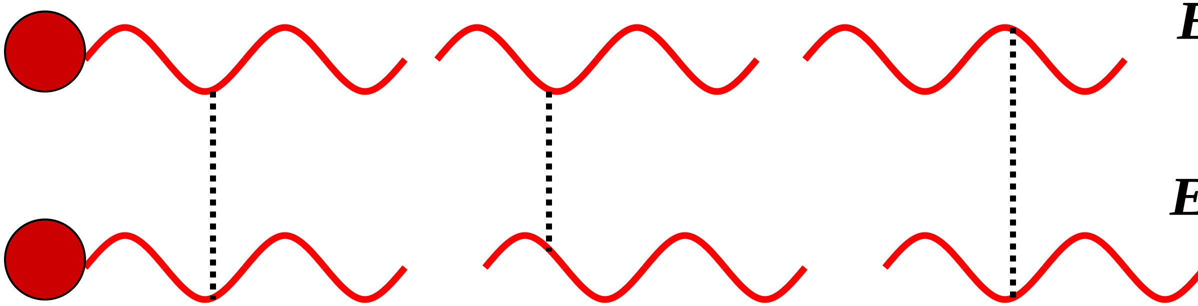
$$\varphi_{x_1} - \varphi_{x_2} = 2\pi m \qquad m = 0, \pm 1, \pm 2 \ldots$$

Destructive Interference: The phase difference between two waves should be π or π integer number of 2π

$$\varphi_{x_1} - \varphi_{x_2} = \pi + 2\pi m \qquad m = 0, \pm 1, \pm 2 \ldots$$

Conditions for Interference

coherent

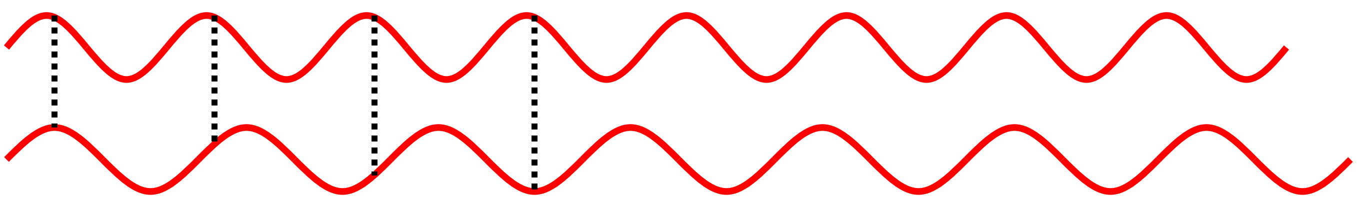


$$E(x) = E_0 \sin(\omega t + \varphi_1)$$

$$E(x) = E_0 \sin(\omega t + \varphi_2)$$

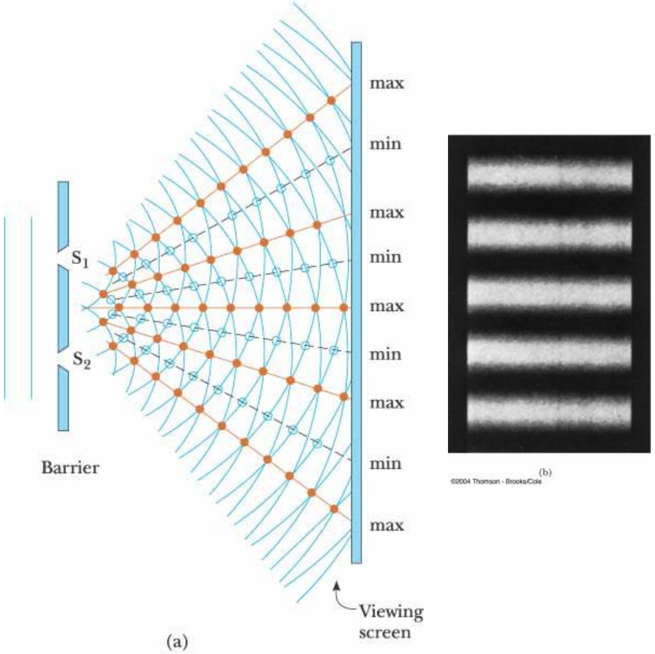
The sources should be monochromatic (have the same frequency)

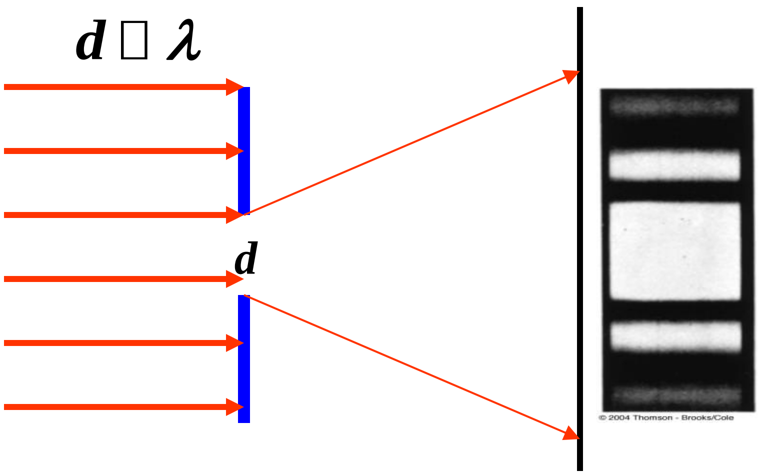
$$E(x) = E_0 \sin(\omega_1 t + \varphi)$$



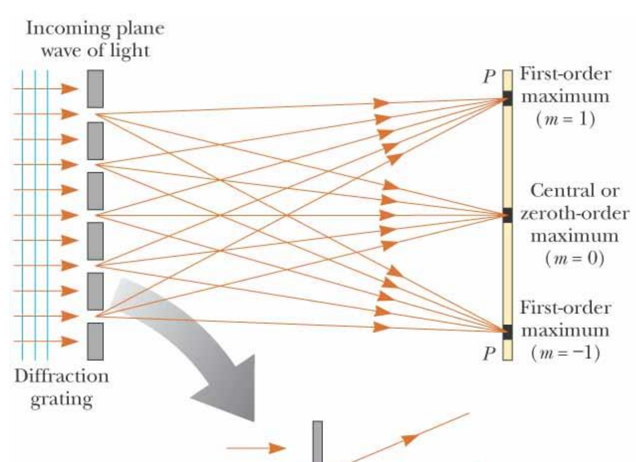
$$E(x) = E_0 \sin(\omega_2 t + \varphi)$$

1. Double-Slit Experiment (interference)





2. Single-Slit Diffraction



3. Diffraction Grading

INTERFERENCE OF LIGHT

IN 1801 YOUNG DEMONSTRATED

① CONSTRUCTIVE AND DESTRUCTIVE EFFECTS OF LIGHT USING DOUBLE-SLIT EXPERIMENT

② LIGHT SOURCE WAS MONOCHROMATIC (SINGLE WAVELENGTH)

③ LIGHT SOURCE NEED TO BE COLLINATED.
USED A DOUBLE SLIT ARRANGEMENT

different effect. Unfortunately, human eyes see a new frame every one-fiftieth of a second (we see 50 frames per second). We are therefore unable to see the interference effects of two separate light sources. Two such natural light sources are said to be incoherent.

Interference between electromagnetic waves can be demonstrated using radio waves or microwaves. In these waves, the electromagnetic radiation is created by physically accelerating electrons in the transmitting aerial using an alternating voltage. The waves in these cases are continuous and therefore maintain a constant phase relationship. The two aerials are linked so they are fed the same signal.

To demonstrate interference effects in light is more difficult. There are two ways of carrying out such experiments. First, a single light source can be used. The light is split into two beams, using slits or mirrors. Each wave packet is split so that it travels to the observer along a different path. If, when the beams are recombined, each individual wave packet combines with its other half, any interference effects can be observed.

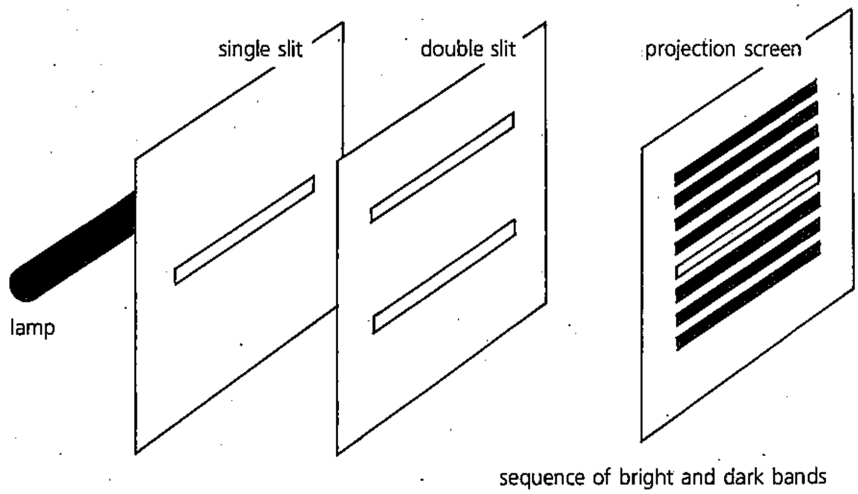
For both halves of the wave packet to arrive at the observer at the same time, the difference in length of the path along which each travels must be less than about 3 m (the length of the wave packet).

The second method uses two lasers as light sources. Lasers are a special form of light source in which the atoms are triggered into emitting wave packets by a synchronising light wave. Each of the wave packets is emitted in phase with the synchronising light wave and therefore form a coherent light source (Figure 12.16 (b)).

Young's double slit experiment

Thomas Young first demonstrated interference between two light waves in 1801. He used a single source of light waves, splitting them at two slits as shown in Figure 12.17.

Figure 12.17
Young's double slit experiment



The following points summarise the main parts of the experimental set-up:

- the light source was a nearly **monochromatic** source, emitting light at a single wavelength. Suitable light sources would be sodium or mercury gas discharge lamps. The experiment could also be done with a filament lamp and a narrow band filter or a laser source

- the single slit allowed light from only one part of the lamp to pass through the system
- the double slits were narrow and aligned along the same direction as the single slit. These slits split each individual wave packet into two separate parts which then travel out from the slits as circular (or cylindrical waves)
- the waves from each slit illuminate the screen and interact to produce an interference pattern.

A given point on the screen lies at slightly different distances from each slit. The waves therefore travel different distances before reaching that point on the screen. The difference between path lengths is known as the **path difference** (*pd*) to that point. As long as the path difference is shorter than the length of the wave packet, interference effects will be observed at the screen.

If the path difference to a point is a whole number of wavelengths, the two waves will arrive in phase and interfere constructively. A bright fringe will be seen at these points. That is, bright fringes occur when:

$$pd = n\lambda \qquad \text{where } n = 0, 1, 2, 3\ldots$$

If the path difference to the point is not a whole number of wavelengths, but a half wavelength, or three half wavelengths, etc., then the waves interfere destructively and we see a dark band at that point. That is, dark fringes occur when:

$$pd = (n + \tfrac{1}{2})\lambda \qquad \text{where } n = 0, 1, 2, 3\ldots$$

On the screen Young observed a series of light and dark bands, parallel to the slit direction. A bright fringe occurs immediately opposite the centre-line of the pair of slits. The separations of the bright fringes were found to depend on the following parameters:

- the distance between the pair of slits—as the slits are moved further apart, the distance between the bright fringes decreases
- the wavelength of the light illuminating the slits—if the wavelength of the light is increased, the distance between the bright fringes increases. The separation for red light is greater than the separation of fringes for blue light
- the distance between the pairs of slits and the viewing screen—as the distance between the slits and the screen is increased, the bright fringes move further apart.

The bright fringes fade in brightness as one moves away from the centre-line toward the top and bottom of the screen as shown in Figure 12.18. Narrowing the slits themselves, while maintaining their separation, makes the fringe pattern more uniformly bright. Widening the slits makes the fringes fade more rapidly with distance from the central fringe. This is due to diffraction effects

Young also noticed that, if the first slit was removed, the fringe pattern disappeared.

The geometry of Young's double slits

The observed effects can be explained only if light is considered as a wave. The key to calculating the interference pattern is the path difference between waves from each slit to points on the screen.

Bright fringes occur only when this path difference is a whole number of wavelengths. What occurs before the double slits plays no part in the calculation, although it does ensure that the light passing through the two sources is coherent.

Figure 12.18
Intensity pattern for Young's double slit experiment

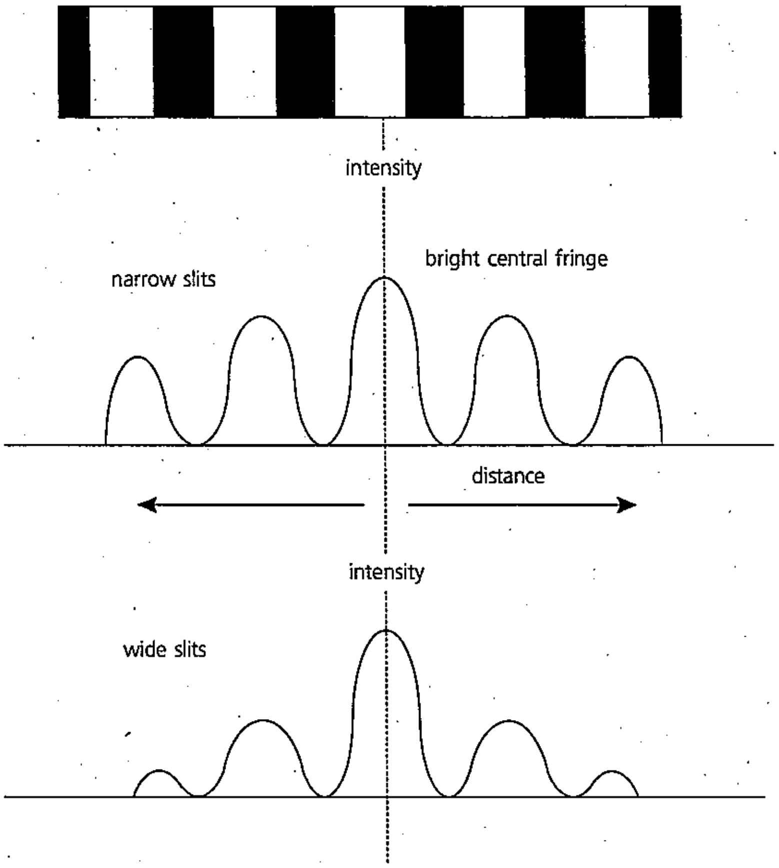
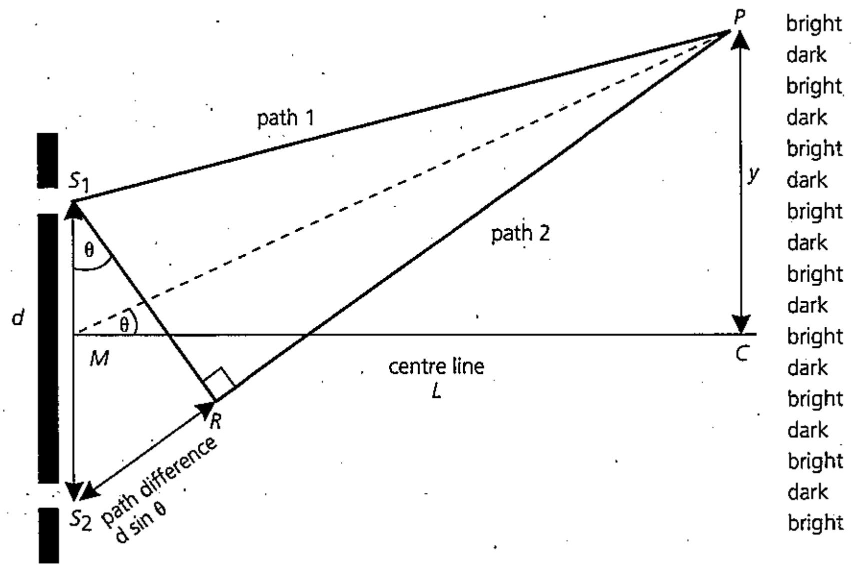


Figure 12.19 illustrates the geometry of the experimental set-up. The distance from the double slits to the screen is L and the slit separation is d. If the distance to the screen is very much greater than the slit separation then the angles CMP and S_2S_1R are the same:

$$\text{angle } CMP = \text{angle } S_2S_1R = \theta$$

$$\tan\theta = \frac{CP}{MC} = \frac{y}{L}$$

Figure 12.19
The geometry of Young's double slit experiment



For small angles:

$$\tan\theta = \theta = \frac{y}{L}$$

and:

$$\sin\theta = \tan\theta = \frac{y}{L}$$

The difference between the path of the wave from S_1 to the point P and the wave from S_2 to P is:

$$pd = S_2R$$

From triangle S_2S_1R, this distance is given by:

$$pd = d\sin\theta$$

For constructive interference between the two waves, this path difference must be equal to a whole number of wavelengths. That is, for a bright fringe:

$$n\lambda = d\sin\theta = \frac{dy}{L} \qquad \text{where } n = 0, 1, 2, 3, 4\ldots$$

And for destructive interference, a dark fringe is obtained when:

$$(n + \tfrac{1}{2})\lambda = d\sin\theta = \frac{dy}{L} \qquad \text{where } n = 0, 1, 2, 3, 4\ldots$$

Bright fringes are expected to occur at all positions where:

$$y = \frac{Ln\lambda}{d}$$

The bright central fringe is represented by the value $n = 0$. The next bright fringe is given by $n = 1$ and occurs at:

$$y_1 = \frac{L\lambda}{d}$$

The next bright fringe is given by $n = 2$ and occurs at:

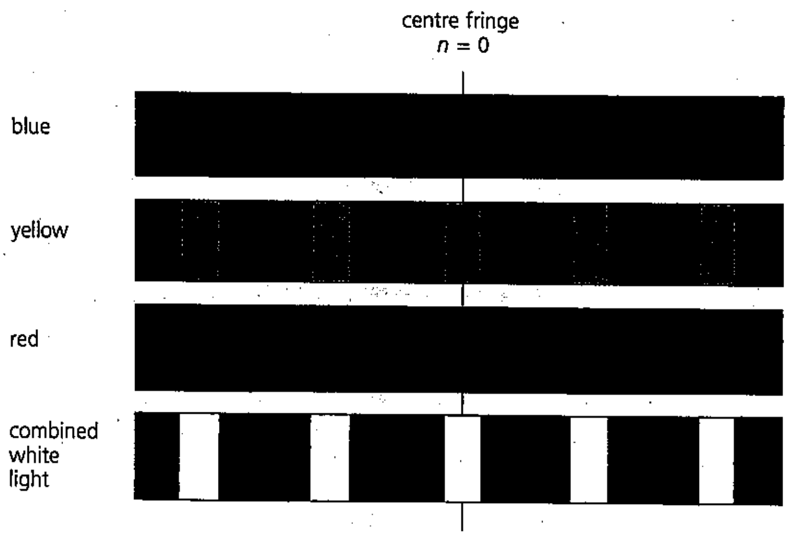
$$y_2 = \frac{2L\lambda}{d}$$

The separation of two adjacent bright fringes is given by the difference:

$$y_2 - y_1 = \frac{L\lambda}{d}$$

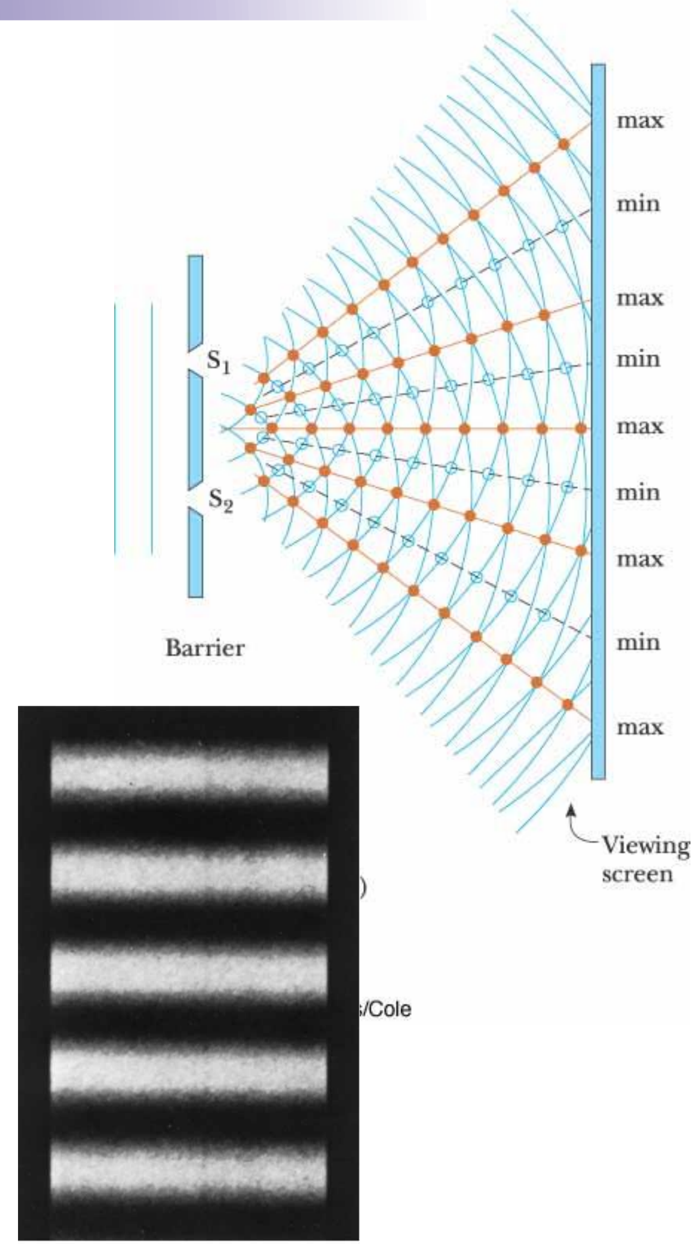
Increasing L or λ causes the fringes to move further apart. Increasing the slit separation d causes the fringes to move closer together.

Figure 12.20
Young's double slit experiment and white light



Young's Double-Slit Experiment

- **Thomas Young first demonstrated interference in light waves from two sources in 1801**
- **The narrow slits S_1 and S_2 act as sources of waves**
- **The waves emerging from the slits originate from the *same wave front* and therefore *are always in phase***



(b)

YOUNG'S EXPERIMENT

$$Y_{BRIGHT} = \frac{L\lambda n}{d}$$

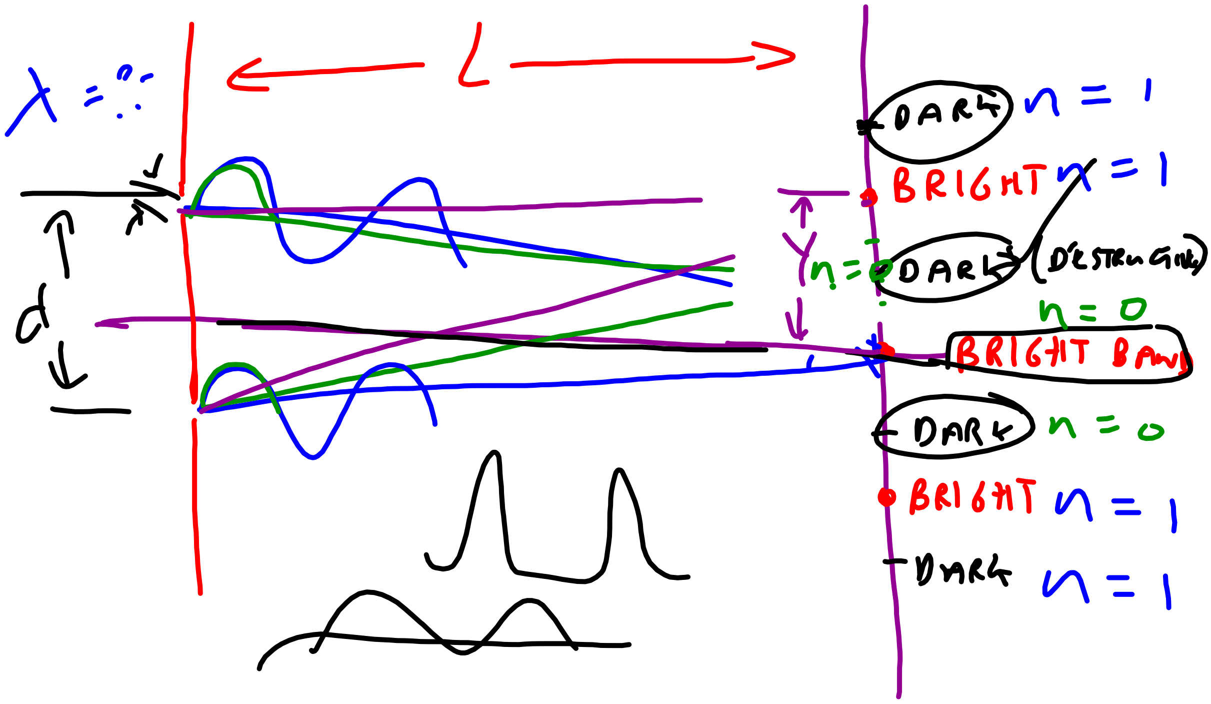
L = LENGTH SCREEN

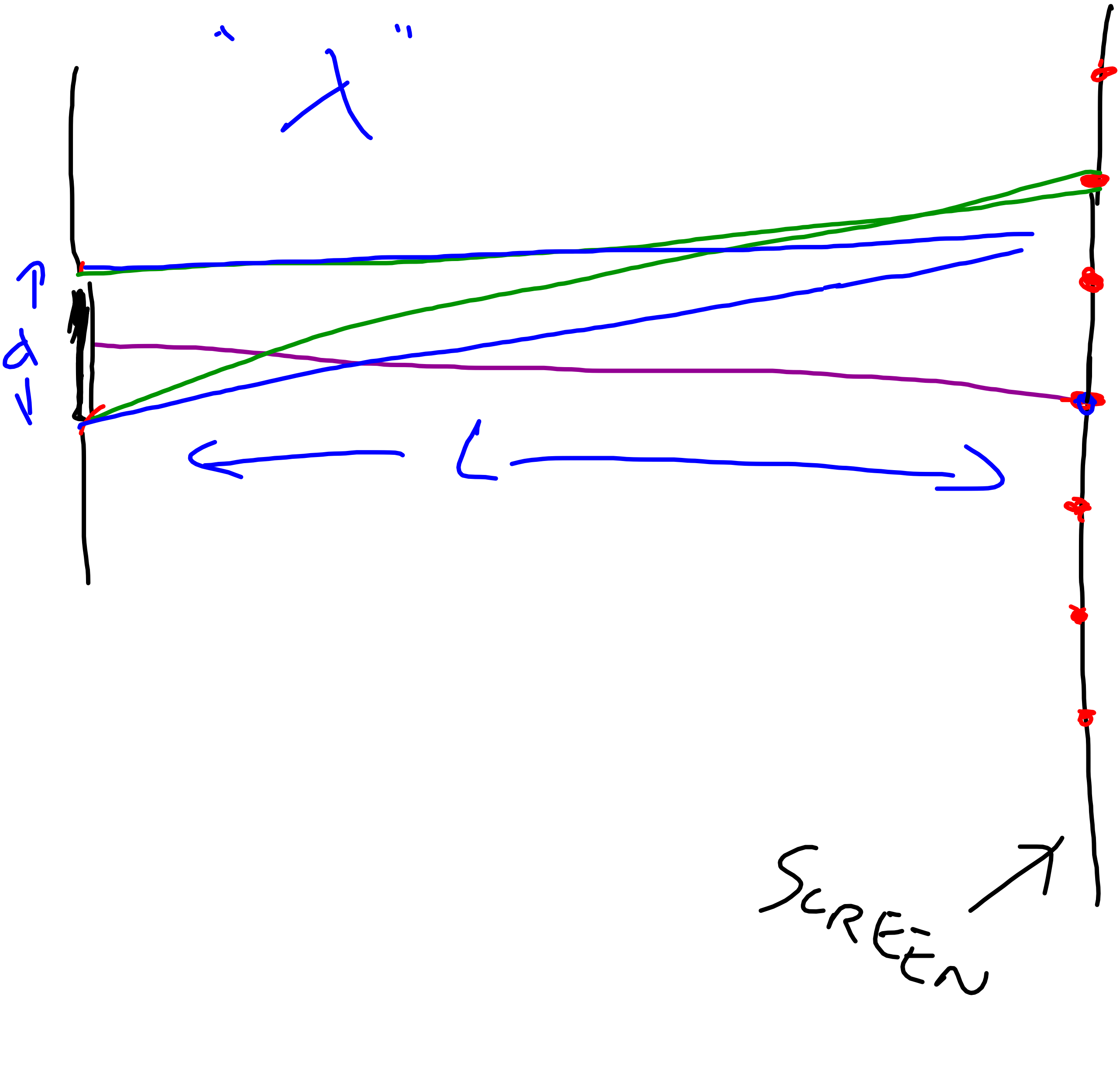
d = SLIT SPACING

n = BAND NUMBER

λ = WAVELENGTH

$$Y_{DARK} = \frac{L\lambda(n+\frac{1}{2})}{d}$$


λ = ?
L
d
Y
DARK n = 1
BRIGHT n = 1
n = 0 DARK (DESTRUCTIVE)
n = 0
BRIGHT BAND
DARK n = 0
BRIGHT n = 1
DARK n = 1


"λ"
d
L
SCREEN

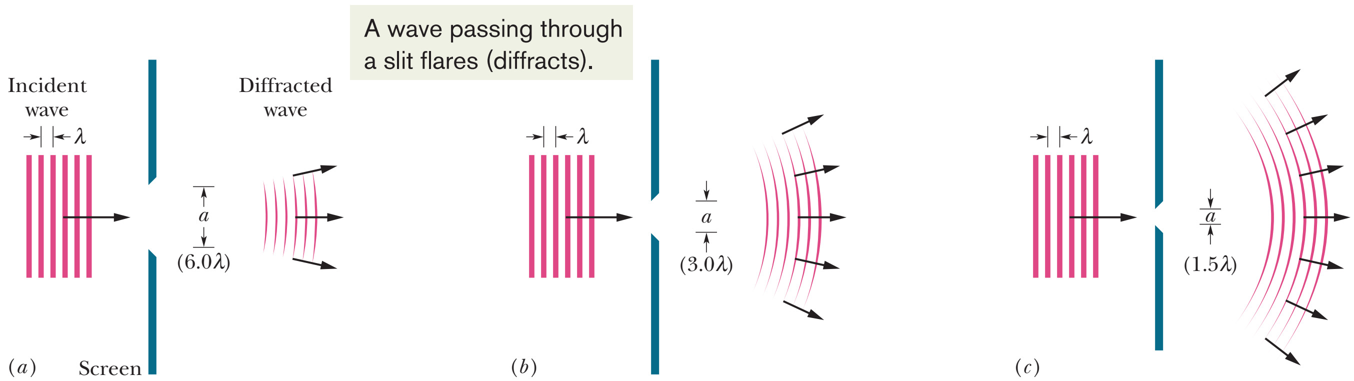


Fig. 35-7 Diffraction represented schematically. For a given wavelength λ, the diffraction is more pronounced the smaller the slit width a. The figures show the cases for (a) slit width $a = 6.0\lambda$, (b) slit width $a = 3.0\lambda$, and (c) slit width $a = 1.5\lambda$. In all three cases, the screen and the length of the slit extend well into and out of the page, perpendicular to it.

Figure 35-7a shows the situation schematically for an incident plane wave of wavelength λ encountering a slit that has width $a = 6.0\lambda$ and extends into and out of the page. The part of the wave that passes through the slit flares out on the far side. Figures 35-7b (with $a = 3.0\lambda$) and 35-7c ($a = 1.5\lambda$) illustrate the main feature of diffraction: the narrower the slit, the greater the diffraction.

Diffraction limits geometrical optics, in which we represent an electromagnetic wave with a ray. If we actually try to form a ray by sending light through a narrow slit, or through a series of narrow slits, diffraction will always defeat our effort because it always causes the light to spread. Indeed, the narrower we make the slits (in the hope of producing a narrower beam), the greater the spreading is. Thus, geometrical optics holds only when slits or other apertures that might be located in the path of light do not have dimensions comparable to or smaller than the wavelength of the light.

35-4 Young's Interference Experiment

In 1801, Thomas Young experimentally proved that light is a wave, contrary to what most other scientists then thought. He did so by demonstrating that light undergoes interference, as do water waves, sound waves, and waves of all other types. In addition, he was able to measure the average wavelength of sunlight; his value, 570 nm, is impressively close to the modern accepted value of 555 nm. We shall here examine Young's experiment as an example of the interference of light waves.

Figure 35-8 gives the basic arrangement of Young's experiment. Light from a distant monochromatic source illuminates slit S_0 in screen A. The emerging light

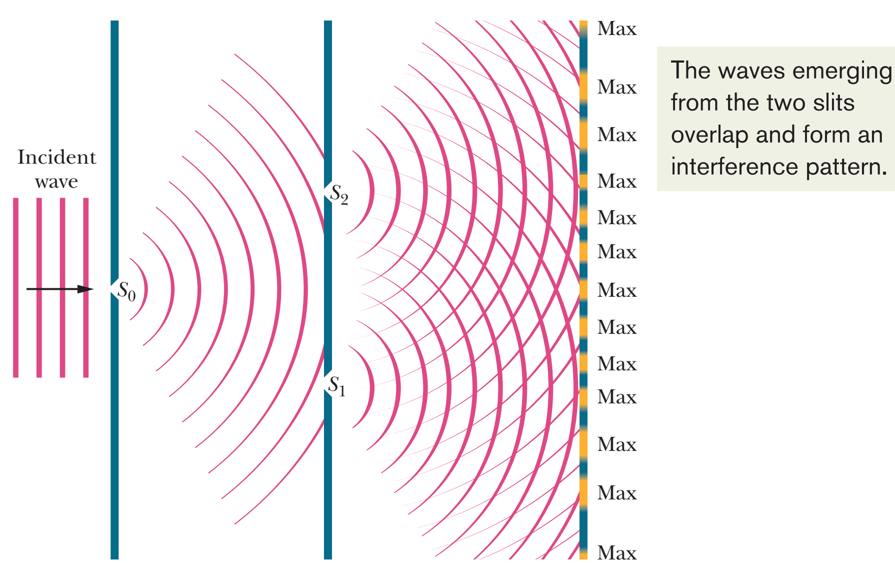


Fig. 35-8 In Young's interference experiment, incident monochromatic light is diffracted by slit S_0, which then acts as a point source of light that emits semicircular wavefronts. As that light reaches screen B, it is diffracted by slits S_1 and S_2, which then act as two point sources of light. The light waves traveling from slits S_1 and S_2 overlap and undergo interference, forming an interference pattern of maxima and minima on viewing screen C. This figure is a cross section; the screens, slits, and interference pattern extend into and out of the page. Between screens B and C, the semicircular wavefronts centered on S_2 depict the waves that would be there if only S_2 were open. Similarly, those centered on S_1 depict waves that would be there if only S_1 were open.

then spreads via diffraction to illuminate two slits S_1 and S_2 in screen B. Diffraction of the light by these two slits sends overlapping circular waves into the region beyond screen B, where the waves from one slit interfere with the waves from the other slit.

The "snapshot" of Fig. 35-8 depicts the interference of the overlapping waves. However, we cannot see evidence for the interference except where a viewing screen C intercepts the light. Where it does so, points of interference maxima form visible bright rows—called *bright bands, bright fringes,* or (loosely speaking) *maxima*—that extend across the screen (into and out of the page in Fig. 35-8). Dark regions—called *dark bands, dark fringes,* or (loosely speaking) *minima*—result from fully destructive interference and are visible between adjacent pairs of bright fringes. (*Maxima* and *minima* more properly refer to the center of a band.) The pattern of bright and dark fringes on the screen is called an **interference pattern.** Figure 35-9 is a photograph of part of the interference pattern that would be seen by an observer standing to the left of screen C in the arrangement of Fig. 35-8.

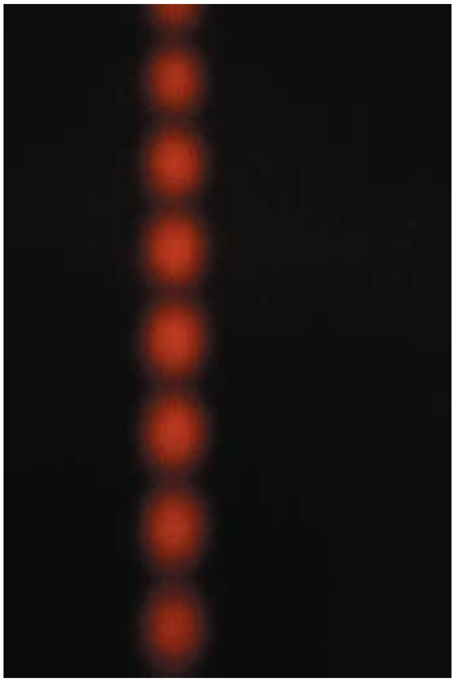


Fig. 35-9 A photograph of the interference pattern produced by the arrangement shown in Fig. 35-8, but with short slits. (The photograph is a front view of part of screen C.) The alternating maxima and minima are called *interference fringes* (because they resemble the decorative fringe sometimes used on clothing and rugs). *(Jearl Walker)*

Locating the Fringes

Light waves produce fringes in a *Young's double-slit interference experiment,* as it is called, but what exactly determines the locations of the fringes? To answer, we shall use the arrangement in Fig. 35-10*a*. There, a plane wave of monochromatic light is incident on two slits S_1 and S_2 in screen B; the light diffracts through the slits and produces an interference pattern on screen C. We draw a central axis from the point halfway between the slits to screen C as a reference. We then pick, for discussion, an arbitrary point P on the screen, at angle θ to the central axis. This point intercepts the wave of ray r_1 from the bottom slit and the wave of ray r_2 from the top slit.

These waves are in phase when they pass through the two slits because there they are just portions of the same incident wave. However, once they have passed the slits, the two waves must travel different distances to reach P. We saw a similar situation in Section 17-5 with sound waves and concluded that

> The phase difference between two waves can change if the waves travel paths of different lengths.

The change in phase difference is due to the *path length difference* ΔL in the paths taken by the waves. Consider two waves initially exactly in phase, traveling along paths with a path length difference ΔL, and then passing through some common point. When ΔL is zero or an integer number of wavelengths, the waves arrive at the common point exactly in phase and they interfere fully con-

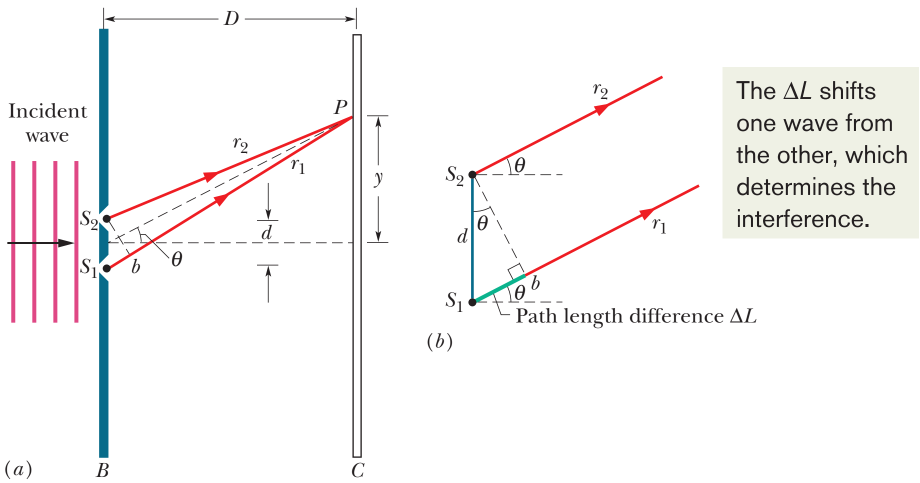


Fig. 35-10 (*a*) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at P, an arbitrary point on screen C at distance y from the central axis. The angle θ serves as a convenient locator for P. (*b*) For $D \gg d$, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

Double-Slit Experiment: Interference

$$E(x) = E_0 \sin(\omega t + \varphi_x)$$

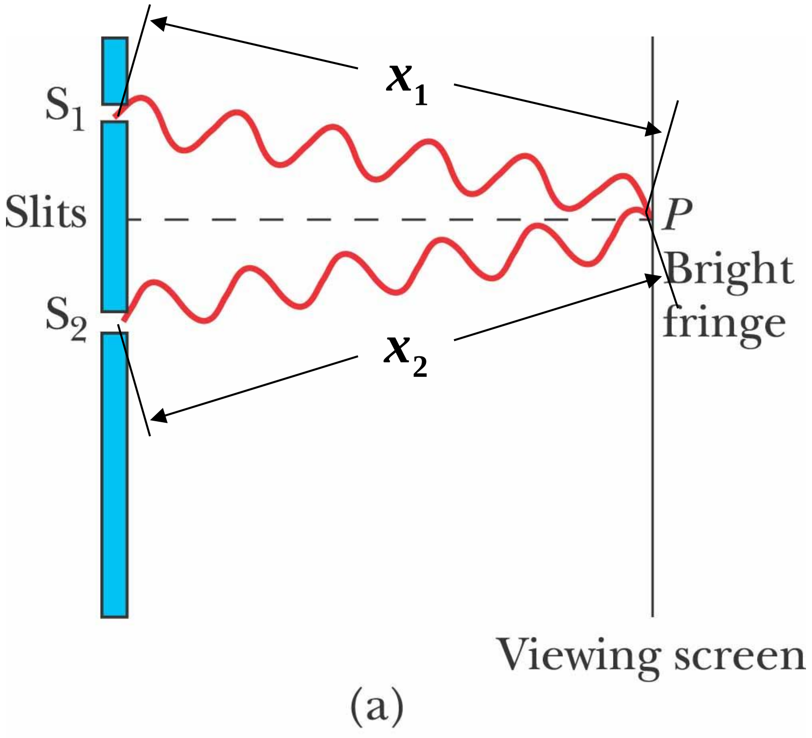
$$\varphi_x = 2\pi \frac{x}{\lambda}$$

The phase of wave 1:

$$\varphi_{x,1} = 2\pi \frac{x_1}{\lambda}$$

The phase of wave 2:

$$\varphi_{x,2} = 2\pi \frac{x_2}{\lambda}$$



Constructive Interference: $\varphi_{x,2} - \varphi_{x,1} = 2\pi n$ where ***n*** is integer
(bright fringe)

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi n \longrightarrow x_2 - x_1 = n\lambda$$

Destructive Interference: $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi n$ where ***n*** is integer
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$

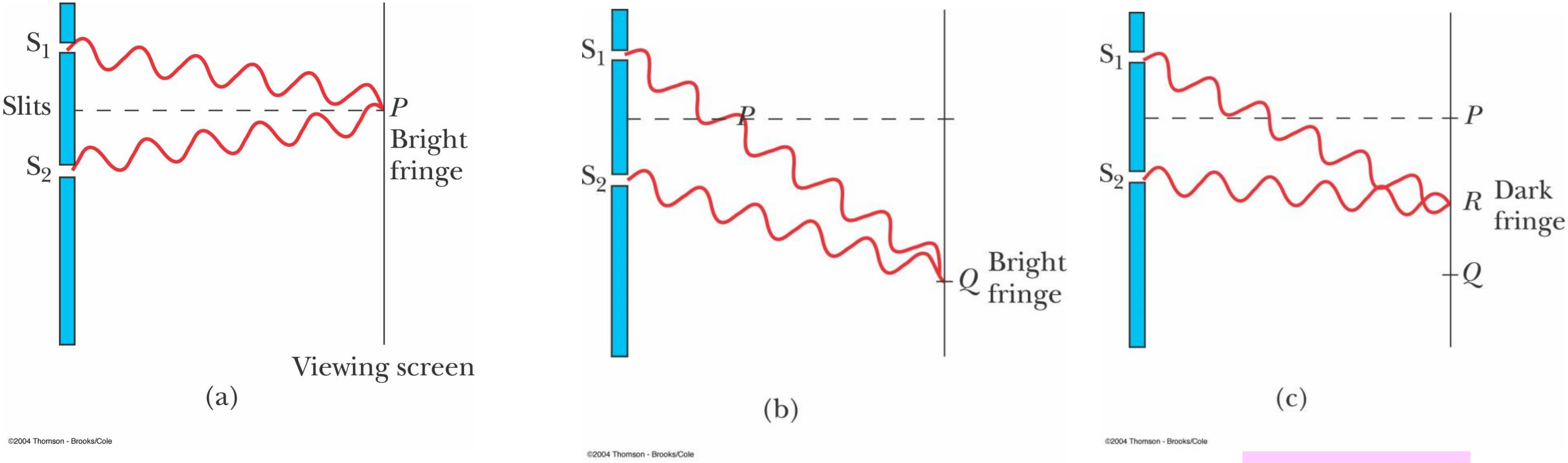
Double-Slit Experiment: Interference

Constructive Interference:
(bright fringe)

$$x_2 - x_1 = n\lambda$$

Destructive Interference:
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$



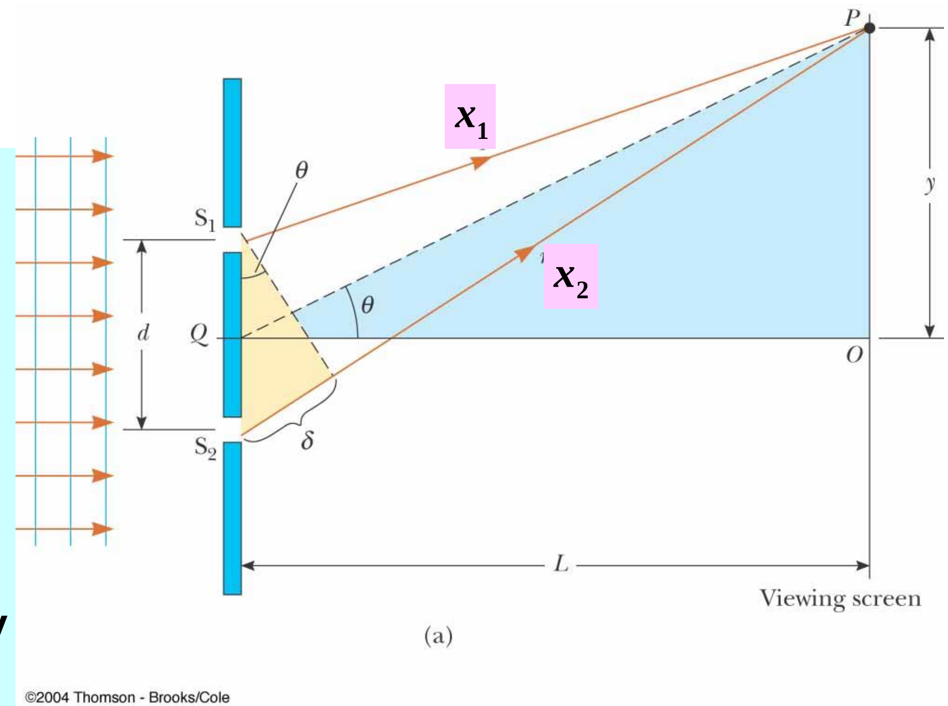
$$x_2 - x_1 = 0$$

$$x_2 - x_1 = \lambda$$

$$x_2 - x_1 = \frac{\lambda}{2}$$

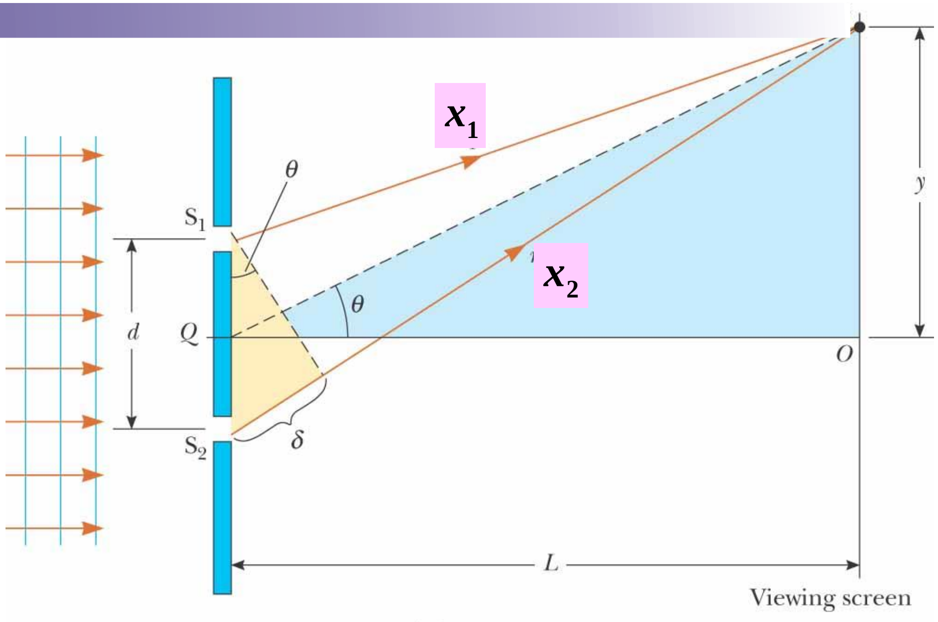
Double-Slit Experiment: Interference

- The path difference, δ, is found from the tan triangle
- $\delta = x_2 - x_1 = d \sin \theta$
  - This assumes the paths are parallel
  - Not exactly true, but a very good approximation if *L* is much greater than *d*



Double-Slit Experiment: Interference

$$\delta = x_2 - x_1 = d \sin \theta$$



(a)

Bright fringes (constructive interference):

$\delta = d \sin \theta = n\lambda$ $\qquad$ $n = 0, \pm 1, \pm 2, \ldots$

n is called the **order number**

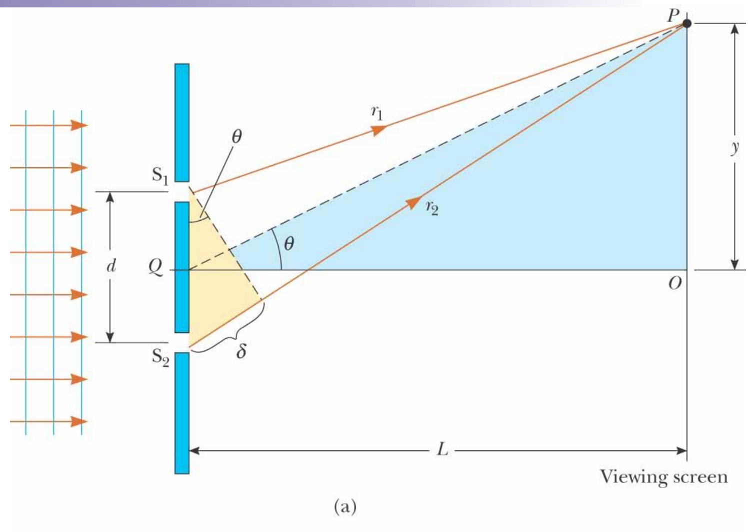
- when $n = 0$, it is the *zeroth-order maximum*
- when $n = \pm 1$, it is called the *first-order maximum*

Dark fringes (destructive interference):

$\delta = d \sin \theta = (n + ½)\lambda$ $\qquad$ $n = 0, \pm 1, \pm 2, \ldots$

Double-Slit Experiment: Interference

$$\delta = x_2 - x_1 = d \sin\theta$$



The positions of the fringes can be measured vertically from the zeroth-order maximum

θ is small and therefore the small angle approximation $\tan\theta \sim \sin\theta$ can be used

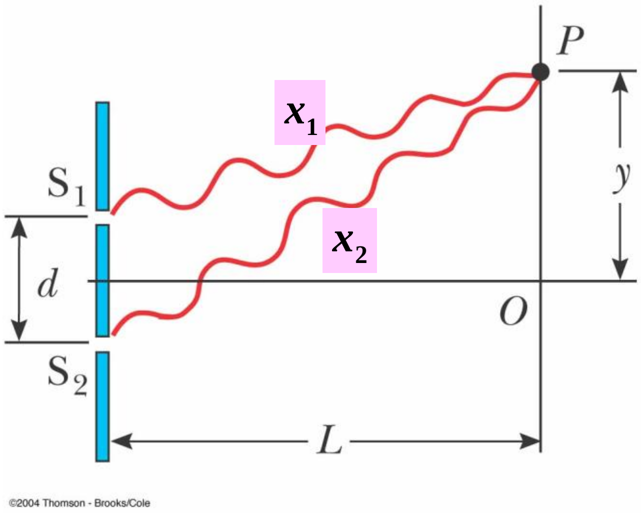
$$y = L\tan\theta \approx L\sin\theta$$

For bright fringes

$$y_{\text{bright}} = \frac{\lambda L}{d} n \quad (n = 0, \pm 1, \pm 2 \ldots)$$

For dark fringes

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$



Constructive Interference: $\varphi_{x,2} - \varphi_{x,1} = 2\pi n$ where ***n*** is integer $n = 0, \pm 1, \pm 2, ...$
(bright fringe)

$$2\pi \frac{x_2}{\lambda} - 2\pi \frac{x_1}{\lambda} = 2\pi n \longrightarrow x_2 - x_1 = n\lambda$$

$$y_{\text{bright}} = \frac{\lambda L}{d} n \quad (\text{n} = 0, \pm 1, \pm 2 \ldots)$$

Destructive Interference: $\varphi_{x,2} - \varphi_{x,1} = \pi + 2\pi n$ where ***n*** is integer $n = 0, \pm 1, \pm 2, ...$
(dark fringe)

$$x_2 - x_1 = \frac{\lambda}{2} + n\lambda$$

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(n + \frac{1}{2}\right) \quad (n = 0, \pm 1, \pm 2 \ldots)$$

YOUNG'S EXPERIMENT

$$Y_{BRIGHT} = \frac{L\lambda n}{d}$$

L = LENGTH SCREEN
d = SLIT SPACING
n = BAND NUMBER
λ = WAVELENGTH

$$Y_{DARK} = \frac{L\lambda(n+\frac{1}{2})}{d}$$

Student Practice Problem:

Q1/ A Young's double slit experiment has a slit spacing of 2mm (d)

screen distance of 40cm (L)

(λ) $\lambda = 600nm$

$$Y_{BRIGHT} = \frac{nL\lambda}{d}$$

$$Y_{DARK} = \frac{(0+\frac{1}{2})L\lambda}{d}$$

① Position of 1st dark band

② Position of 2nd bright band away from central

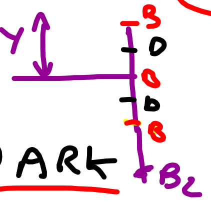
Remember:

Central Maxima Bright band refers to n = 0

Q1// A YOUNG'S DOUBLE SLIT EXPERIMENT HAS A SLIT SPACING OF 2 mm (d)

SCREEN DISTANCE OF 40 cm (L)

(λ) $\lambda = 600\,nm$



$$Y_{BRIGHT} = \frac{nL\lambda}{d}$$

$$Y_{DARK} = \frac{(0+\frac{1}{2})L\lambda}{d}$$

1. POSITION OF 1st DARK BAND
2. POSITION OF 2ND BRIGHT BAND AWAY FROM CENTRAL

$\lambda = 600\times10^{-9}\,m$

$L = 40\times10^{-2}\,m$

$d = 2\times10^{-3}\,m$

$n = 0$

$$Y_{DARK} = \frac{(n+\frac{1}{2})L\lambda}{d}$$

$$= \frac{\frac{1}{2}\times40\times10^{-2}\times600\times10^{-9}}{2\times10^{-3}}$$

$$= 60\,\mu m$$

② $Y_{BRIGHT} = \frac{n\lambda L}{d}$

$n = 2$

$$= \frac{2\lambda L}{d} = \frac{2\times40\times10^{-2}\times600\times10^{-9}}{2\times10^{-3}}$$

$$= 240\,\mu m = 0.24\,mm$$

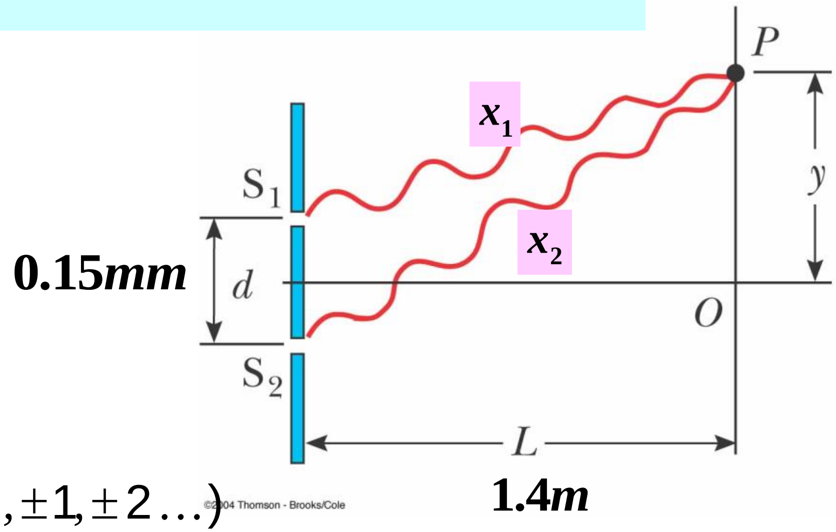
Double-Slit Experiment: Example

The two slits are separated by 0.150 mm, and the incident light includes light of wavelengths $\lambda_1 = 540nm$ and $\lambda_2 = 450nm$. At what minimal distance from the center of the screen the bright line of the λ_1 light coincides with a bright line of the λ_2 light

Bright lines:

$$y_{bright,1} = \frac{\lambda_1 L}{d} n_1 \quad (n_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{bright,2} = \frac{\lambda_2 L}{d} n_2 \quad (n_2 = 0, \pm 1, \pm 2 \ldots)$$



$$y_{bright,1} = \frac{540 \cdot 10^{-9} \cdot 1.4}{015 \cdot 10^{-3}} n_1(\text{m}) = 5n_1(\text{mm}) \qquad (n_1 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{bright,2} = \frac{450 \cdot 10^{-9} \cdot 1.4}{015 \cdot 10^{-3}} n_2(\text{m}) \approx 4n_2(\text{mm}) \qquad (n_2 = 0, \pm 1, \pm 2 \ldots)$$

$$y_{bright,1} = 0, 5, 10, 15, 20, 25 \ldots (\text{mm})$$

$$y_{bright,1} = 0, 4, 8, 12, 16, 20 \ldots (\text{mm})$$

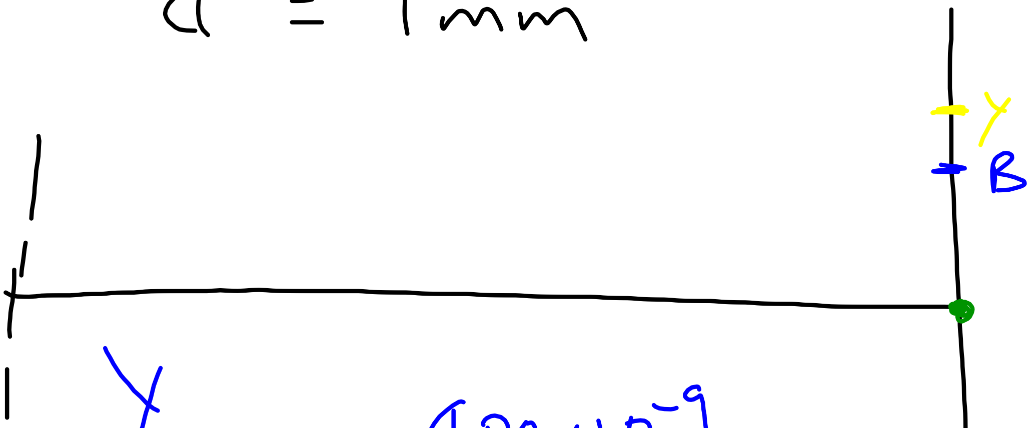
BLUE = 400nm

YELLOW = 500nm

$Y = \frac{nL\lambda}{d}$

L = 100cm

d = 1mm



$Y_{B(n=1)} = \frac{400\times10^{-9}\times1}{1\times10^{-3}}$

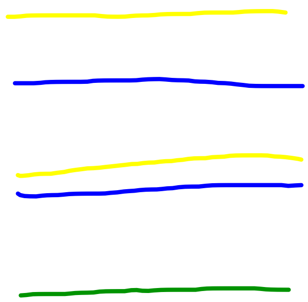
$= 0.4\,mm$

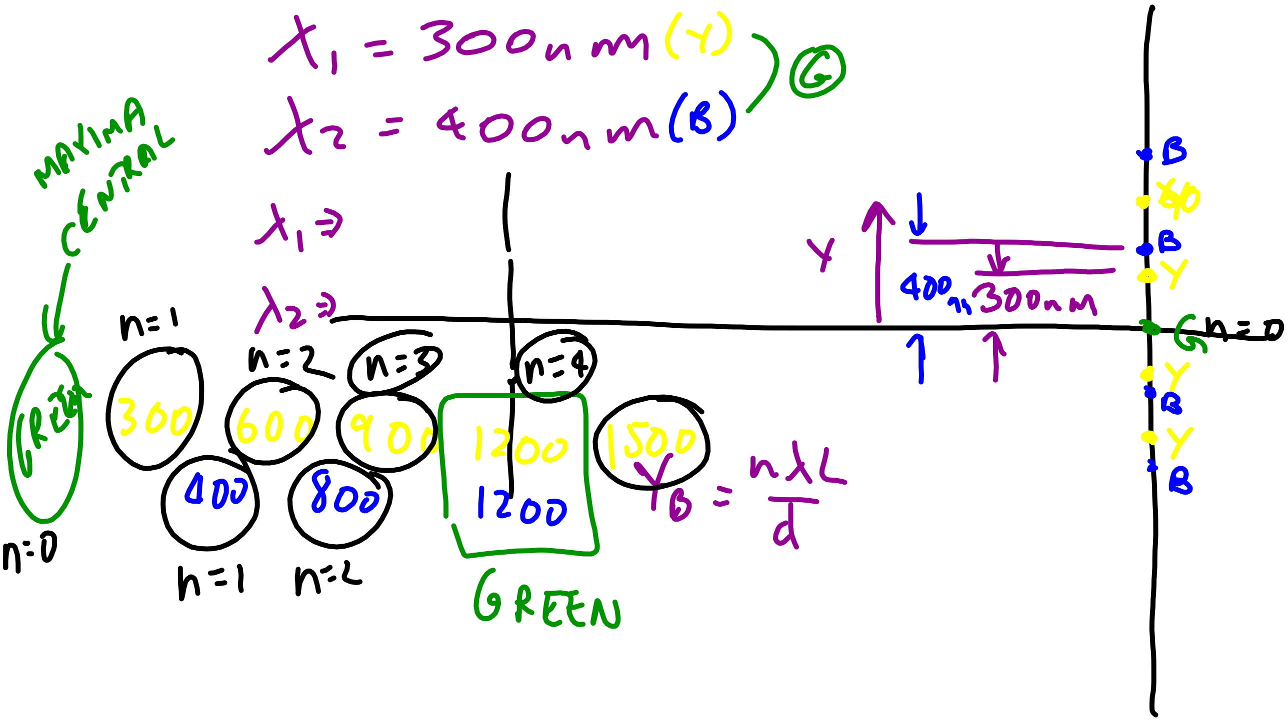
$Y_{Y(n=1)} = 0.5\,mm$

CENTRAL G | 0.4 B | 0.8 B | 1.2 B | 1.6 B | 2.0 G

Y (0.5) | Y (1.0) | Y (1.5)

↑ MIXTURE




λ1 = 300nm (Y)
λ2 = 400nm (B)
G
MAXIMA CENTRAL
GREEN
n=0
λ1 ⇒
λ2 ⇒
n=1
n=2
n=3
n=4
300
600
900
1200
1500
400
800
1200
n=1
n=2
GREEN
YB = nλL / d
Y
400nm
300nm
B
Y
G
n=0

Q2 A YOUNG'S EXPERIMENT HAS 2 LIGHT SOURCES. YELLOW OF FREQUENCY = 5×10^{14} Hz
PURPLE OF FREQUENCY = 6×10^{14} Hz

IF SLIT SPACING = 3mm AND SCREEN DISTANCE = 6m

$v_{Light} = 3 \times 10^{8} ms^{-1}$
$= c$

FIND (A) COLOUR OF THIRD BRIGHT BAND AWAY FROM CENTRAL MAXIMA

(B) DISTANCE TO FIRST BAND OF COLOUR MIXTURE (YELLOW + PURPLE) AWAY FROM CENTRAL MAXIMA

$$v = f\lambda$$

$$v = f\lambda \quad \therefore \lambda = \frac{v}{f}$$

$$= \frac{3 \times 10^8}{}$$

YOUNG'S EXPERIMENT

$$Y_{BRIGHT} = \frac{L\lambda n}{d}$$

L = LENGTH SCREEN
d = SLIT SPACING
n = BAND NUMBER
λ = WAVELENGTH

$$Y_{DARK} = \frac{L\lambda(n+\frac{1}{2})}{d}$$

ANSWER

$$\lambda_{YELLOW} = \frac{c}{f} = \frac{3\times10^{8}}{5\times10^{14}} = 600\,nm$$

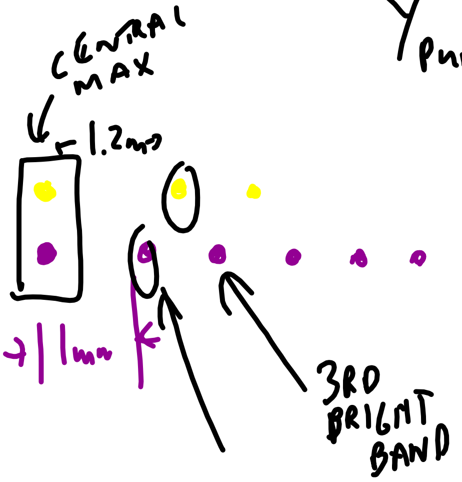
$$\lambda_{PURPLE} = \frac{c}{f} = \frac{3\times10^{8}}{6\times10^{14}} = 500\,nm$$

$$Y_{BRIGHT} = \frac{L\lambda n}{d}$$

$(n=1)$

$$Y_{YELLOW} = \frac{600\times10^{-9}\times6}{3\times10^{-3}} = 1.2\,mm$$

$$Y_{PURPLE} = \frac{500\times10^{-9}\times6}{3\times10^{-3}} = 1.0\,mm$$



YELLOW: 0, 1.2, 2.4, 3.6, 4.8, 6.0

PURPLE: 0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0

ANSWER TO (1): COLOUR OF THIRD BAND = PURPLE

TO Q2: MIXTURE OCCURS AT 6mm